EADR: Entropy Adjusted Dynamic Routing Capsule Networks

Reza Babaei
School of Electrical and Computer Engineering
The University of Oklahoma
Norman, United States
rezababaei@ou.edu

Huong Pham
School of Electrical and Computer Engineering
The University of Oklahoma
Norman, United States
huongnpham@ou.edu

Samuel Cheng
School of Electrical and Computer Engineering
The University of Oklahoma
Norman, United States
samuel.cheng@ou.edu

Shangqing Zhao
School of Computer Science
The University of Oklahoma
Norman, United States
shangqing@ou.edu

2025 59th Annual Conference on Information Sciences and Systems (CISS) | 979-8-3315-1326-9/25/$31.00 ©2025 IEEE | DOI: 10.1109/CISS64860.2025.10944763

***Abstract*—Capsule networks offer a useful approach for modeling part-whole hierarchies in visual data, yet they remain limited in effectively handling uncertainty in part-object relationships. This work introduces an entropy-adjusted dynamic routing (EADR) algorithm that leverages information-theoretic principles to enhance both the performance and interpretability of capsule networks. By incorporating an entropy-based regularization term into the final iteration of the dynamic routing process, our approach refines routing decisions, reducing reliance on uncertain capsule connections. Experimental evaluations on the CIFAR-10 dataset demonstrate that our method achieves a mean accuracy of 85.73%, surpassing the baseline accuracy of 84.22% achieved by standard dynamic routing. Comparative analysis with recent entropy-based routing methods highlights our approach's balance of computational efficiency and routing flexibility, achieved without much additional model complexity. These results suggest that EADR routing can be a useful tool for enhancing both the interpretability and efficacy of capsule networks in uncertain environments.**

***Index Terms*—Capsule Networks, Information Theory, Dynamic Routing, Entropy Adjustment**

I. Introduction

Substantial evidence suggests that humans interpret visual scenes by organizing them into part-whole hierarchies, modeling the spatial relationships between components and their wholes as viewpoint-invariant coordinate transformations between intrinsic coordinate frames [7], [12]. Inspired by this human capability, researchers have sought to replicate these mechanisms computationally [3]. This is the foundational objective of capsule networks, which are specifically designed to capture these hierarchical relationships explicitly [8]–[10], [13], [16]. Capsules are groups of artificial neurons that act as vector-valued units, encapsulating multiple properties of entities such as pose, color, and size. They represent specific features or entities within a network, with their exact definition varying depending on their roles and connection mechanisms.

The Vice President for Research and Partnerships of the University of Oklahoma, the Data Institute for Societal Challenges, and the Stephenson Cancer Center through DISC/SCC Seed Grant Award

The "routing-by-agreement" mechanism in traditional capsule networks, known as dynamic routing [17], enables capsules to communicate and adapt their connections based on the consistency or "agreement" between their outputs. This dynamic routing process is designed to preserve equivariance, allowing the network to recognize objects regardless of changes in orientation or position within the input space. Through dynamic routing, the agreement between lower-level capsule outputs and higher-level capsule predictions filters out irrelevant features, emphasizing only those that contribute meaningfully to object understanding. This high-dimensional filtering enhances both feature detection and representation learning.

Research has underscored the importance of dynamic routing's iterative nature in fostering part-whole hierarchies critical for object classification [20]. However, the conventional dynamic routing algorithm in capsule networks overlooks uncertainties in the routing process, potentially leading to suboptimal performance [15]. While some recent works have introduced entropy as an uncertainty measure in capsule connection protocols or as an auxiliary term in the loss function to encourage certainty-driven voting [4], [5], [15], [19], these methods do not integrate entropy into the core iterative routing process.

This study introduces a novel approach by embedding entropy directly into the dynamic routing process, transforming entropy from a peripheral metric into a core component that guides routing decisions. By harnessing information already present within the routing process, our method enhances both model performance and interpretability without adding much additional complexity. This entropy-adjusted approach represents a step forward in making the voting process inherently more resilient, interpretable, and responsive to uncertainty, marking an advancement in both the performance and interpretability of capsule networks.

II. Related Work

Recent advancements have applied entropy to capsule networks to enhance their routing mechanisms, improving both network interpretability and performance. De et al. [4] introduced a methodology to compute the average negative entropy for quantifying vote agreement, thereby capturing the consensus of the capsule's pose parameters. Capsule activation is modulated by the level of agreement, with higher agreement leading to increased activation. The authors employ a Binomial distribution to model the expected support each capsule receives, facilitating a probabilistic evaluation of capsule activations based on vote agreement and support. This work establishes a foundation for interpreting capsule activation through an entropy-informed perspective.

This concept was extended in the Variational Bayes (VB) routing algorithm proposed by Ribeiro et al. [15], where entropy is utilized to evaluate the agreement among votes from lower-level capsules. The method incorporates the differential entropy of a higher capsule's Gaussian-Wishart variational posterior distribution to measure vote alignment. Higher entropy indicates greater uncertainty in alignment, providing a means to quantify uncertainty within the variational distribution. By adjusting capsule activations based on this measure, the VB routing algorithm improves both performance and interpretability, offering insights into how capsules agree on vote patterns.

Building on their earlier work [20], Venkataraman et al. [19], proposed EntrLoss, a novel loss function designed to enhance compositionality within capsule networks. EntrLoss leverages the entropy of the routing weights between layers, encouraging more decisive, parse-tree-like routing decisions. By minimizing entropy, the loss function promotes a dominant routing weight per capsule connection, enhancing compositional representation learning. This selective routing mechanism results in superior performance in classification tasks compared to recent capsule models.

Last but not least, Renzulli et al. [14] introduced the Routing Entropy Minimization (REM) method, which improves the interpretability of capsule networks through quantization and pruning after training. REM reduces the entropy of coupling coefficients between capsules, fostering a parse-tree-like structure in the routing process. By promoting sparsity and minimizing variability in routing, this technique clarifies the relationships among capsules, resulting in a more explainable CapsNet. The selective and stable routing paths enforced by REM enhance the transparency of decision-making within the network.

III. Methodology

Recently, Venkataraman et al. [19] reaffirmed the foundational role of routing in capsule networks, as originally envisioned, and emphasized its effectiveness in fostering part-whole hierarchies under specific conditions. However, traditional routing algorithms often result in high entropy in their routing weights, which can undermine the part-to-whole hierarchy learning capabilities of these networks [20]. Venkataraman et al. [20] argue that reducing entropy facilitates more accurate modeling of part-whole hierarchies.

A. Review of dynamic routing

The dynamic routing mechanism in capsule networks tackles the challenge of part-to-whole by iteratively adjusting routing coefficients between lower-level and higher-level capsules. These adjustments are guided by the agreement between their output vectors, enabling the network to establish structured relationships between parts and wholes. In capsule layers $l+1$, the capsule j's non-squashed vote, $s_{:,j}$, is a weighted sum of prediction vectors $\hat{u}_{j|i}$, computed as:

$$s_{:,j} = \sum_i c_{i,j} \hat{u}_{j|i}, \tag{1}$$

$$\hat{u}_{j|i} = W_{i,j} u_i, \tag{2}$$

where u_i is the output of a capsule in the layer l, $W_{i,j}$ is the weight matrix, and $c_{i,j}$ are coupling coefficients determined by dynamic routing as explained below.

The coupling coefficients $c_{i,j}$ between capsule j and all capsules in the layer l sum to 1 and are computed using a routing Softmax. The initial logits $b_{i,j}$ represent the log prior probabilities of coupling capsule i with capsule j, which can be learned alongside other weights. The initial coupling coefficients are refined iteratively by measuring the agreement $a_{i,j}$ between the output $v_{:,j}$ of capsule j and the prediction $\hat{u}_{j|i}$ from lower level capsules:

$$a_{i,j} = v_{:,j} \cdot \hat{u}_{j|i}. \tag{3}$$

This agreement, treated as a log-likelihood, is added to $b_{i,j}$ to update the coupling coefficients.

B. EADR Routing

To address the limitations of traditional dynamic routing, we propose an enhanced algorithm that incorporates entropy as a central element of the routing process. Entropy, as a measure of uncertainty, offers a systematic and efficient means to guide routing decisions, enabling the network to dynamically adjust routing coefficients based on the certainty of capsule connections. This integration is achieved by introducing an entropy regularization term during the final routing iteration, providing a principled approach to refining the routing process.

We introduce an entropy term to regulate the routing process. The entropy term quantifies the uncertainty in the routing decisions. First, we calculate the entropy $H(c_{:,j})$ of the routing coefficients $c_{i,j}$, derived from the Softmax function, which measures the uncertainty in the routing. The detailed modification incorporating this entropy term is highlighted in Algorithm 1.

$$H(c_{:,j}) = -\sum_i c_{i,j} \log(c_{i,j} + \epsilon), \tag{4}$$

Algorithm 1 EADR Routing Algorithm

ROUTING ($\hat{u}_{j|i}$, r, l , λ, ϵ)
for all capsule i in layer l and capsule j in layer (l+1) :
Initialize $b_{i,j} = 0$
for r iterations **do**
  for all capsule j in layer $l+1$:
    $c_{:,j} = \text{Softmax}(b_{:,j})$
    $s_{:,j} = \sum_i c_{i,j}\hat{u}_{j|i}$
  if r = final iteration **then**
    $H(c_{:,j}) = -\sum_i c_{i,j}\log(c_{i,j}+\epsilon)$
    $s_{:,j} = \sum_i c_{i,j}\hat{u}_{i|j} - \lambda H(c_{:,j})$
  end if
    for all capsule j in layer (l+1): $v_{:,j} = \frac{||s_{:,j}||^2}{1+||s_{:,j}||^2}\frac{s_{:,j}}{||s_{:,j}||}$
    for all capsule i in layer l and capsule j in layer (l+1):
    $a_{i,j} = \hat{u}_{j|i} \cdot v_{;,j}$
    $b_{i,j} = b_{i,j} + a_{i,j}$
  end for
  Return: Higher-layer capsules $v_{:,j}$

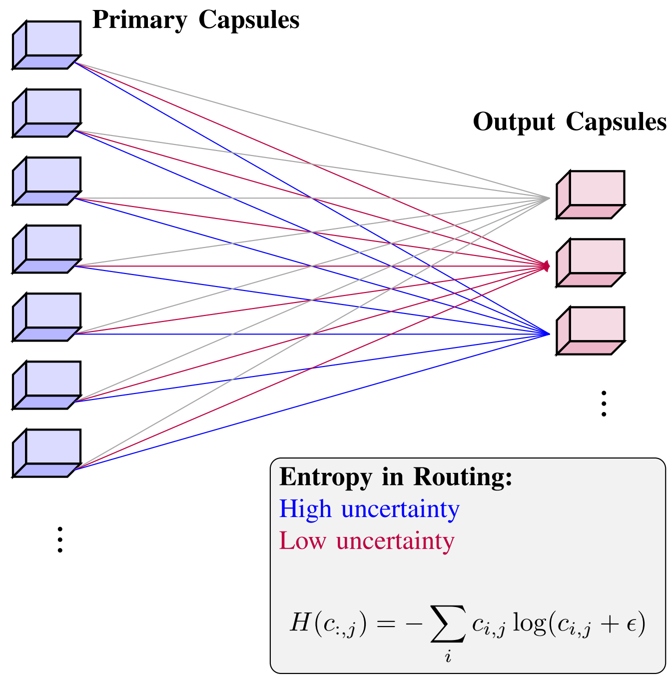

Fig. 1: Dynamic Routing **Before** Entropy Adjustment

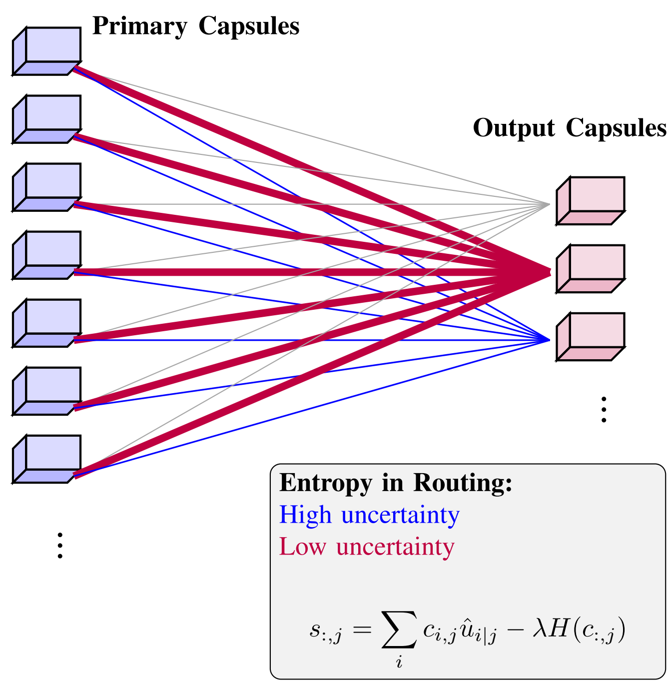

Fig. 2: Dynamic Routing **After** Entropy Adjustment

where ϵ is a small constant added for numerical stability to avoid taking the logarithm of zero. The entropy term $H(c_{:,j})$ captures the degree of uncertainty in the routing: higher entropy indicates greater uncertainty, while lower entropy indicates more confident routing decisions.

We modify the routing process by incorporating the entropy into the prediction updates. The updated prediction vector is computed by subtracting the entropy integration term from the routing coefficients $c_{i,j}$ times the input predictions $\hat{u}_{j|i}$:

$$s_{:,j} = \sum_i c_{i,j}\hat{u}_{j|i} - \lambda H(c_{:,j}), \tag{5}$$

where entropy coefficient λ is a hyperparameter that controls the strength of the entropy integration. This term reduces the contribution of uncertain routes (those with higher entropy), effectively encouraging more confident routing decisions. This regularization is designed to be both efficient and scalable, requiring no modifications to the network's core architecture or the addition of extensive new parameters. As a result, the proposed approach introduces minimal computational overhead while delivering noticeable improvements in performance and interpretability.

In summary, the addition of the entropy term introduces a bias toward more confident routing decisions. Without entropy integration, the routing coefficients $c_{i,j}$ are highly sensitive to small changes in the logits, which can lead to instability when multiple routes have similar logits, resulting in high uncertainty. By incorporating the entropy term, we penalize uncertain predictions, i.e., when the Softmax distribution $c_{i,j}$ is more spread out (indicating high entropy). This forces the routing process to favor more certain predictions (lower entropy), reducing instability caused by uncertain routing decisions.

IV. Experiments

We performed evaluations on our proposed modification to verify its effectiveness in practical settings as it will be explored in the following section.

A. Experimental Setup

We evaluate the performance of our proposed EADR routing algorithm using the CIFAR-10 dataset. The CIFAR-10 dataset comprises 60,000 color images of size 32x32, divided into 10 distinct classes. Among these, 50,000 images are used for training, and the remaining 10,000 images serve as test samples, providing a well-balanced foundation for evaluating classification performance. The model was trained for 300 epochs to ensure sufficient learning and stability in convergence, using a batch size of 64 to balance computational efficiency with model accuracy. The entropy coefficient λ was initialized at -0.001. Throughout the training, it was gradually learned to stabilize the routing process, balancing between model predictions and the entropy-based dynamic routing. The modification introduces a minor loop only in the last routing iteration, adding negligible computational cost.

B. Results Analysis

Table I summarizes the performance of our EADR routing model in comparison to the baseline dynamic routing model, based on multiple experimental runs. The mean accuracy of our method reached 85.73%, an improvement over the baseline accuracy of 84.22%. This increase demonstrates the effectiveness of entropy integration in enhancing generalization capabilities and the robustness of the routing mechanism, particularly for complex class distinctions.

TABLE I: Comparison of CIFAR-10 Test Accuracy

Method	**Accuracy (%)**
Dynamic Routing [17]	84.22
EM Routing [10]	82.19
Inverted Dot-Product Attention Routing [18]	85.17
VB-Routing [15]	83.80
REM [14]	79.25
EADR routing	**85.73**

As training advanced, λ, initially set to zero, stabilized around a value of +0.005, which emerged as the optimal setting for maximizing model performance based on the CIFAR-10 dataset. This adaptive adjustment of entropy facilitated more effective routing without excessive restrictions during early training. The convergence of λ is observed as shown in the Figure 3

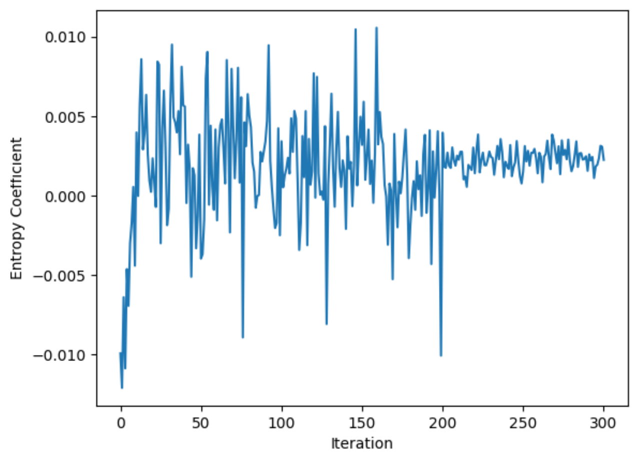

Fig. 3: Entropy coefficient λ during 300 epochs of training

V. Discussion

Incorporating entropy into the dynamic routing process plays a crucial role in improving the efficiency and interpretability of the capsule network. In this context, entropy serves as a regularization mechanism that encourages stronger child-parent capsule connections when there is high certainty and reduces overly confident connections when certainty is low. By integrating this concept, the routing process becomes more informed, balancing the certainty of routing decisions with the need for flexibility in ambiguous cases.

Our experiments indicate that allowing dynamic routing to function freely during initial iterations, followed by incorporating entropy adjustments in the final iteration, yields superior performance compared to applying entropy from the outset. Initially, dynamic routing benefits from exploring potential connections between child and parent capsules, refining coupling coefficients based on initial votes. This approach enables the network to establish foundational relationships without encountering the over-polarization issues often observed in the final iterations.

The use of entropy, which is an interpretable concept from information theory, not only enhances the efficiency of the routing process but also provides insight into how capsules interact and make decisions. By maximizing the effectiveness of the iterations through entropy, the model incorporates more meaningful knowledge into the routing process while maintaining computational efficiency. Thus, entropy acts as a bridge between maximizing the information flow in the network and ensuring the interpretability of the decision-making process within the dynamic routing framework.

A. Comparing EADR versus Existing Approaches

Unlike our dynamic routing-based approach that directly integrates entropy into the routing process, Ribeiro et al. [15] introduced the Variational Bayes (VB) routing algorithm, which utilizes a Bayesian probabilistic framework, modeling uncertainty over capsule parameters and routing weights.

In the work by Ribeiro et al. [4], the authors introduce a method that utilizes negative entropy to quantify vote agreement in capsule networks, emphasizing the certainty of capsule poses. They compute the vote certainty as the negative entropy $-h(M_j)$ of a Gaussian distribution $\mathcal{N}(M_j|\mu_j, \sigma_j)$ fitted during training, where M_j denotes the pose matrix or vector of the object capsule j in a capsule network, while μ_j and σ_j represent the mean and standard deviation of the capsule votes. The EADR mechanism adjusts activations, favoring capsules with low entropy, which indicates high vote agreement and facilitates consistent pose assignment, although, the performance of the proposed model was not evaluated on the CIFAR-10 dataset.

Furthermore, some studies specifically integrate entropy into the loss function, as seen in the EntrLoss method introduced by Venkataraman et al. [19]. This approach aims to enhance compositionality by minimizing the mean entropy of the routing weights for each shallower capsule type, defined as: :

$$\text{EntrLoss} = \frac{1}{N_l|G|} \sum_{g \in G} \sum_{i=0}^{N_{l-1}} \left(-c_{i,j}^{l+1}(g) \log(c_{i,j}^{l+1}(g)) \right),$$

where N_l and N_{l+1} are the number of capsules in layers l and $l+1$, respectively, g is a group of capsules, and G is the set of all groups being considered for routing.

And finally, the Routing Entropy Minimization (REM) technique [14] reduces the entropy of coupling coefficients in the routing mechanism by introducing pruning. After training, REM extracts parse trees by quantizing the coupling coefficients, organizing them into dictionaries, and measuring entropy to analyze the hierarchical relationships between capsules, resulting in interpretable representations of the model's learned structures. However, they encountered worse classification accuracies on the CIFAR-10 and Tiny ImageNet datasets, attributing this to noise introduced in their coupling coefficients, which led to increased uncertainties in the classification task.

TABLE II: Comparing with other capsule networks leveraging entropies

Methods	Description/Key differences
EADR (ours)	Routing coefficients are adjusted using non-parametric entropy values through dynamic agreement calculations between capsules
VB routing [15]	This work uses the differential entropy of a capsule's variational posterior distribution as a measure of agreement instead of the scalar products of capsule outputs and activates capsules only after completing the routing iterations, rather than during iterative routing
Uncertainty capsule routing [4]	This work employs a global probabilistic framework based on variational inference rather than local iterative routing between adjacent capsule layers
EntrLoss [19]	This work incorporated entropy as a term in the loss function instead of modifying the iterative routing protocol. It was evaluated across multiple capsule connection layers rather than a single connection distribution
REM [14]	This work leverages entropy to prune the network tree rather than modifying routing rule

B. Limitations & Future Directions

The findings of this study highlight several limitations and avenues for future research, particularly concerning the computational efficiency of capsule networks. While various efficient versions of capsule routing have been proposed [1], [6], [15], [18], the challenges extend beyond routing to encompass the capsule voting procedure. As identified by Barham et al. [2], current computational frameworks are heavily optimized for a narrow range of operations typical in established model architectures. This optimization leads to inefficiencies when handling the unique computational demands posed by routing and voting procedures in capsule networks, resulting in slower performance for non-standard workloads.

Future research should focus on optimizing capsule network architectures to improve compatibility with existing computational frameworks and hardware accelerators [11]. Additionally, exploring the potential of specialized hardware designed for capsule networks may yield further enhancements in computational efficiency and model scalability.

VI. Conclusion

In this study, we proposed an EADR routing algorithm for capsule networks, designed to address the inherent uncertainty in part-whole relationships that traditional routing approaches struggle to capture. By incorporating an entropy regularization term during the final iteration of dynamic routing, our method dynamically adjusts routing coefficients based on connection certainty, achieving a more balanced and interpretable routing process. Experimental results on CIFAR-10 validate the effectiveness of our approach, with notable improvements in both accuracy and interpretability compared to baseline models.

This entropy-based method offers distinct advantages over prior routing techniques, particularly in tasks requiring high interpretability and efficient handling of complex visual hierarchies. Future work will focus on extending the application of EADR routing to other domains, such as multimodal data tasks, where interpretability and uncertainty handling are paramount. Additionally, optimizing the entropy regularization dynamically across multiple capusle layers may provide further improvements in the adaptability and scalability of capsule networks.

References

[1] K. Ahmed and L. Torresani. Star-caps: Capsule networks with straight-through attentive routing. *Advances in neural information processing systems*, 32, 2019.

[2] P. Barham and M. Isard. Machine learning systems are stuck in a rut. In *Proceedings of the Workshop on Hot Topics in Operating Systems*, pages 177–183, 2019.

[3] T. Cohen and M. Welling. Group equivariant convolutional networks. In *International conference on machine learning*, pages 2990–2999. PMLR, 2016.

[4] F. De Sousa Ribeiro, G. Leontidis, and S. Kollias. Introducing routing uncertainty in capsule networks. *Advances in neural information processing systems*, 33:6490–6502, 2020.

[5] J. Edstedt. Towards understanding capsule networks, 2020.

[6] T. Hahn, M. Pyeon, and G. Kim. Self-routing capsule networks. *Advances in neural information processing systems*, 32, 2019.

[7] G. Hinton. Some demonstrations of the effects of structural descriptions in mental imagery. *Cognitive Science*, 3(3):231–250, 1979.

[8] G. Hinton. How to represent part-whole hierarchies in a neural network. *Neural Computation*, 35(3):413–452, 2023.

[9] G. E. Hinton, A. Krizhevsky, and S. D. Wang. Transforming auto-encoders. In *Artificial Neural Networks and Machine Learning–ICANN 2011: 21st International Conference on Artificial Neural Networks, Espoo, Finland, June 14-17, 2011, Proceedings, Part I 21*, pages 44–51. Springer, 2011.

[10] G. E. Hinton, S. Sabour, and N. Frosst. Matrix capsules with em routing. In *International conference on learning representations*, 2018.

[11] S. Hooker. The hardware lottery. *Communications of the ACM*, 64(12):58–65, 2021.

[12] D. Kahneman, A. Treisman, and B. J. Gibbs. The reviewing of object files: Object-specific integration of information. *Cognitive psychology*, 24(2):175–219, 1992.

[13] A. Kosiorek, S. Sabour, Y. W. Teh, and G. E. Hinton. Stacked capsule autoencoders. *Advances in neural information processing systems*, 32, 2019.

[14] R. Renzulli, E. Tartaglione, and M. Grangetto. Rem: Routing entropy minimization for capsule networks. *arXiv preprint arXiv:2204.01298*, 2022.

[15] F. D. S. Ribeiro, G. Leontidis, and S. Kollias. Capsule routing via variational bayes. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 3749–3756, 2020.

[16] A. Roy, M. Saffar, A. Vaswani, and D. Grangier. Efficient content-based sparse attention with routing transformers. *Transactions of the Association for Computational Linguistics*, 9:53–68, 2021.

[17] S. Sabour, N. Frosst, and G. E. Hinton. Dynamic routing between capsules. *Advances in neural information processing systems*, 30, 2017.

[18] Y.-H. H. Tsai, N. Srivastava, H. Goh, and R. Salakhutdinov. Capsules with inverted dot-product attention routing. *arXiv preprint arXiv:2002.04764*, 2020.

[19] S. R. Venkataraman, S. Balasubramanian, A. Anand, and R. R. Sarma. Learning compositional capsule networks. *Sādhanā*, 49(3):215, 2024.

[20] S. R. Venkatraman, A. Anand, S. Balasubramanian, and R. R. Sarma. Learning compositional structures for deep learning: Why routing-by-agreement is necessary. *arXiv preprint arXiv:2010.01488*, 2020.