

Systolic array implementation for fuzzy evidential reasoning systems

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Keywords: Fuzzy reasoning, Systolic arrays, Possibility theory, Evidential reasoning, Dedicated architectures and Fuzzy hardware.

Abstract: This paper suggests a systolic arrays implementation of fuzzy expert systems based on fuzzy evidential reasoning methodology. Evidential reasoning systems are compute-bound in general and fuzzy systems match larger number of rules than in a simple symbolic reasoning systems. The fuzzy evidential system proposed which is a combination of both needs much more computation than either of these two independently. To speed up processing in such systems the suggested implementation is useful.

1. Introduction.

Evidential reasoning systems will have a set of decisions as consequents as opposed to a single decision in rule-based expert systems. Given a hypothesis and a set of facts, the evidential reasoning systems try to assign upper and lower bounds of credibility to the hypothesis induced by the given facts through the rules in the system. In [10] this methodology has been extended to fuzzy sets by allowing the facts, rules and hypothesis to be fuzzy sets and a dual possibilistic evidential measures to assign possibility bounds to hypothesis induced by fuzzy facts through fuzzy rules. But this method takes higher computation time due to long fuzzy processing functions and computations for partially matched rules.

Recently several ways to speed up inferencing in reasoning systems has been suggested [2-5]. Some of these works are specifically meant for simple fuzzy implication. In Togai et.al., [5] and Manzoul et. al., [2] they have used simple min-relational composition and max-min inference. Systolic architectures of [3] use simple And & Or operations on certainty factors. Other hardware architectures [4, 6] are specific to given applications. In this paper a general purpose systolic array architecture for possibilistic evidential reasoning has been proposed. The implementation provided here uses fuzzy implication as an instruction. This can be changed as necessary and deals with more complex fuzzy evidential reasoning systems. Evidential reasoning systems are compute-bound in general [12] and fuzzy systems match larger number of rules than in a simple symbolic reasoning systems. The fuzzy evidential system proposed which is a combination of both needs much more computation than either of these two independently.

Fuzzy evidential reasoning approach used here is a backward chaining method, which enables all the rules whose consequents have a match or partial match with the given hypothesis fuzzy set based on a similarity measure [11]. Once chains are established from the hypothesis to the facts through the rules, fuzzy inferencing operations will compute the possibility distributions induced by the facts on to the hypothesis space. The terminology and reasoning model used are summarised in the following section. The forward computation of possibility distributions

This work was partially supported by an Australian Research Council grant.

is implemented using systolic arrays in this paper.

2. Fuzzy evidential reasoning model

The reasoning model can be visualised as a disc of rings with a target disc at the centre. Each ring of the disc is made up of several frames of mutually exclusive and exhaustive set of elements. These frames are the frames of discernment in the evidential reasoning model. The outermost ring is the evidence space which absorbs the facts from the outer environment. The target disc contains frames on which the reasoning model draws its inference. Let its frames be called target frames. The directed arcs (edges) start from a frame in an outer ring and connect to a frame in an inner ring, which corresponds to the rules of reasoning. Figure 3. shows the evidential reasoning model for the fuzzy washing machine evidential reasoning model as an example.

Let $S = \{E_1, E_2, \dots\}$ be an evidence space, a set of frames for facts and some rule antecedents, corresponding to the evidence ring of the fuzzy evidential reasoning model. Let $T = \{T_1, T_2, \dots\}$ be a reasoning space, a set of frames for hypotheses and some rule consequents, corresponding to the target disc of the evidential reasoning model. Let $I_j = \{I_{j1}, I_{j2}, \dots\}$ be a set of frames corresponding to the j -th inner ring in the reasoning model. Let $\Phi(X)$ be a set of all fuzzy sets on a frame X .

A reasoning chain is an element of the relation \mathcal{R} defined as

$$\mathcal{R} = \{ (\Phi(E_i) \times \Phi(I_{jp})) \times (\Phi(I_{jp}) \times \Phi(I_{kq})) \dots (\Phi(I_{lr}) \times \Phi(I_{ms})) \times (\Phi(I_{ms}) \times \Phi(T_n)) \} \quad (1)$$

The joint possibility distribution Δ of a reasoning chain, $((A,B), (C,D), \dots, (O,P), (Q,R))$, where $A \in \Phi(E_i), B, C \in \Phi(I_{jp}), \dots, P, Q \in \Phi(I_{ms})$ and $R \in \Phi(T_n)$, is defined as follows:

$$\Delta_{R/A} = (\Pi_{B/A}, \Pi_{D/C} \dots, \Pi_{P/O}, \Pi_{R/Q}) \quad (2)$$

Now given the possibility distribution on the frames of the evidence space a set of basic possibility distributions (*bpdfs*)

$\{\gamma_{ij} \mid \gamma_{ij} \in \Phi(T_i)\}$ can be induced on the target disc T_i through the reasoning chains as a fuzzy composition of the evidence (facts) possibility distribution and Δ of the reasoning chain as follows:

$$\gamma_{ij} = \Pi_E \circ \Delta_{M/N} = (((\Pi_E \circ \Pi_1) \circ \Pi_2) \circ \Pi_3) \dots \quad (3)$$

where $M \in \Phi(T_i), E, N \in \Phi(E_i), \circ$ is Sup-Min composition [8] and $(\Pi_1, \Pi_2, \Pi_3, \dots)$

$= \Delta_{M/N}$ as defined in (2).

In line with the notation of evidential reasoning all non-empty γ_{ij} 's are fuzzy focal elements of frame T_i . Now the credibility bounds of any fuzzy hypothesis $H \in \Phi(T_i)$ can be obtained from the formulae of fuzzy belief (fb) (a lower bound) and fuzzy plausibility (fp) (an upper bound) [10] as follows:

$$\text{fb}(H) = \text{Max}_D \{ \text{Nec}(H; D) \} \quad \text{for all } \text{core}(D) \subseteq H \quad (4)$$

$$\text{fp}(H) = \text{Max}_D \{ \text{Poss}(H; D) \} \quad \text{for all } \text{core}(D) \cap H \neq \emptyset \quad (5)$$

Where D s are fuzzy focals of frame T_i , \cap and \subseteq are fuzzy set intersection and subset operations respectively, $\text{core}(X) = \{x \mid x \text{ is a full member of } X\}$ and

$$\text{Poss}(X; Y) = \text{Sup}_{\Omega} \{ \text{Min}(X(x), Y(x)) \} \quad \text{for all } x \in \Omega \quad (6)$$

$$\text{Nec}(X; Y) = \text{Inf}_{\Omega} \{ \text{Max}(X(x), \neg Y(x)) \} \quad \text{for all } x \in \Omega \quad (7)$$

where Ω is reference set of fuzzy sets X and Y and \neg is fuzzy complement operation.

3. Systolic array architecture for the fuzzy evidential reasoning (FER) model

Different types of Multiprocessor architectures like Multiple-bus, Ring, Hypercube, etc., could be used to implement the FER system described in section 2. Multiple-bus, Ring and Hypercube are ideal for implementation of general purpose algorithms. Systolic arrays are suitable for implementing dedicated algorithms having a regular structure.

The systolic architecture was first proposed by Kung [1] in 1982. The architecture is very suitable for a large class of regular and symmetric algorithms, like matrix multiplication. A systolic architecture consists of a large number of processing elements connected together using an interconnection network which reflects the flow of data among the processing elements. The crux of the architecture is that once a data item has been retrieved from memory, it should be used by all the processing elements which require it. This helps alleviate the processor-memory communication bandwidth problem experienced even by the fastest Von-neumann machine. Systolic architectures reduce the complexity of the algorithms by exploiting the

regularity in the algorithm. They can therefore be applied to speed up the processing in FER systems.

Implementing fuzzy evidential reasoning essentially needs implementing equations (3) to (7). Reasoning process modules implement equation (3). It has the evidence $\{E_i, E_j, \dots, E_k\}$ from the set of evidences and the matching reasoning chain $\{R_x, R_y, \dots, R_z\}$ from the set of chains as inputs. From the inputs it computes the *bpds* (γ),

$\{\pi_{E_i} \circ \pi_{R_x}, \pi_{E_j} \circ \pi_{R_y}, \dots, \pi_{E_k} \circ \pi_{R_z}\}$, as a composition of evidence possibility distribution and joint possibility distribution of the reasoning chains. The systolic arrays for this module are shown in Figure 1.

For illustration, we have shown the details of one reasoning process module having possibility distributions of evidence and hypothesis (target) sets defined on a reference set of cardinality three. Each reasoning process consists of three systolic arrays each of which produces an element of *bpds*, γ . The function carried out (and hence the instructions executed) in each of the systolic arrays is exactly the same as that of module computing the Possibility, P-modules, described later except the logic channel is disabled. Equations (6) and (7) are implemented in the P and N

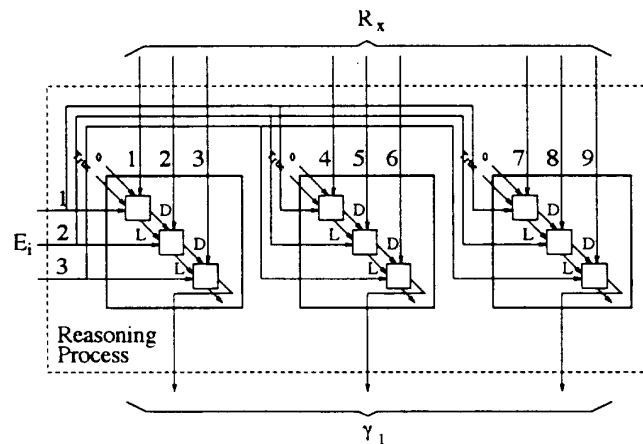


Figure 1. Systolic array implementation of reasoning process

modules respectively as shown in Figure 2. The instructions executed in the processor elements (PE) are shown in Table 1. Each PE inside P has two inputs from the preceding PE. It also has an element of the possibility distributions of the hypothesis (H) and $bpds, \gamma$, as inputs. Each PE computes the minimum of the elements of the H and γ inputs and then finds the maximum of this minimum and the data (D) coming from the preceding PE. The values in the logic channel (L) propagates conditions required in equations (4) and (5). The output of the data channel (D) of P and N contains the results of the equations (6) and (7) respectively, if the conditions in (4) and (5) are satisfied or else the results will be 0.

The output of P and N are fed to two maximum finding modules to compute the Plausibility and Belief. The implementation of the maximum finding modules using systolic arrays is widely available in the literature. From

the $bpds, \{\gamma\}$ and the given hypothesis H_q systolic arrays P_1, \dots, P_m and N_1, \dots, N_m compute the Possibility and Necessity from equations (4) and (5). The outputs of the systolic arrays are fed to two different maximum finding units to compute the Plausibility and Belief as shown in (6) and (7). Each processing element inside P_i and N_i has four input channels (three data channels and one logic channel) and one output channel. The instructions executed in the processing elements are shown in Figure 2.

4. Fuzzy washing machine reasoning with fuzzy evidential model

For testing the evidential reasoning model architecture a fuzzy washing machine with three rules for selecting the washing powder quantity given the

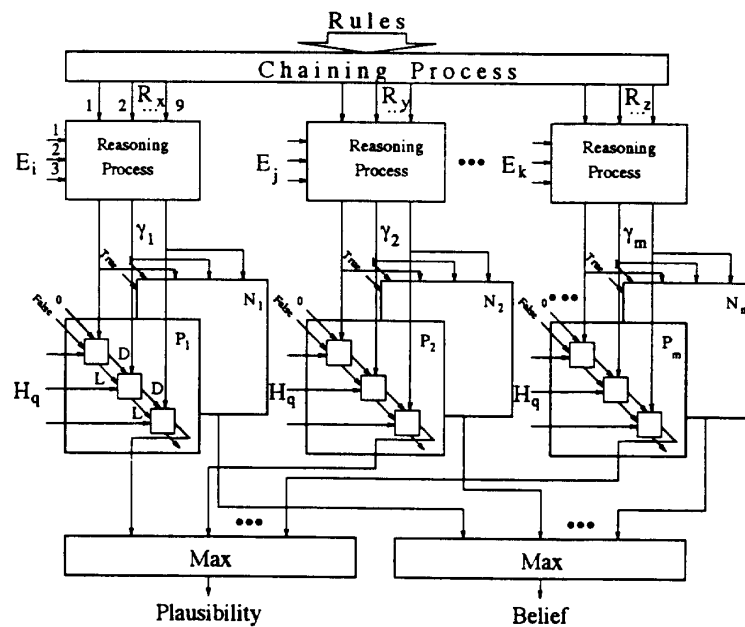


Figure 2. Systolic array implementation of fuzzy evidential reasoning system

N-processor element's instructions:	P-processor element's instructions:
Instruction 1: If $\gamma_1(x) < 1.0$ then temp ₁ (x) = 0.0 else temp ₁ (x) = 1.0.	Instruction 1: If $\gamma_1(x) < 1.0$ then temp ₁ (x) = 0.0 else temp ₁ (x) = 1.0.
Instruction 2: If $H_d(x) \neq \text{temp}_1(x)$ then temp-log ₁ (x) = true else temp-log ₁ (x) = false.	Instruction 2: If $(\text{Min}(H_d(x), \text{temp}_1(x)) \neq 0.0)$ then temp-log ₁ (x) = true else temp-log ₁ (x) = false.
Instruction 3: temp-max ₁ (x) = MAX (H _d (x), 1.0 - $\gamma_1(x)$)	Instruction 3: temp-min ₁ (x) = MIN (H _d (x), $\gamma_1(x)$).
Instruction 4: If (temp-log ₁ (x) AND in-log) then out-min = MIN (in-min, temp-max ₁ (x)) out-log = true else out-min = 0.0 out-log = false.	Instruction 4.1: (all units except last one) If (temp-log ₁ (x) OR in-log) then out-max = MAX (in-min, temp-min ₁ (x)) out-log = true else out-max = MAX (in-min, temp-min ₁ (x)) out-log = false.
	Instruction 4.2: (for last P-processor element only) If (temp-log ₁ (x) OR in-log) then out-max = MAX (in-min, temp-min ₁ (x)) else out-max = 0.0.

Table 1. Instructions for N and P processor elements

dirt level is considered. The fuzzy rules are as follows:

A: IF dirt **low** THEN use washing powder **around 40 to 60 grams**.

B: IF dirt **medium** THEN use washing powder **around 70 to 110 grams**.

C: IF dirt **high** THEN use washing powder **nearly 200 grams**.

Where the bold faced fuzzy predicates in antecedents and consequents of the rules are fuzzy sets defined on dirt on a hypothetical scale and washing powder in grams scale.

Given a fact that dirt is '**between low and medium**' the credibilities derived for hypotheses for washing powder (40 to 60 grams, 30 to 70 grams, 70 to 110 grams, 50 to 130 grams) are as follows:

Hypothesis	Credibilities
40 to 60 grams (H ₁)	[1, 0.05]
30 to 70 grams (H ₂)	[1, 0.65]

70 to 110 grams (H₃) [1, 0.05]

50 to 130 grams (H₄) [1, 1].

It can be observed from the derived credibilities that the system hypothesis that includes more of both consequents of Rule A and Rule B are more credible than the others, as the given fact lies between the antecedents of these rules.

5. Conclusions

Fuzzy evidential reasoning systems are highly compute bound. Different specialised architectures have been suggested for simple *reasoning systems*. Systolic arrays are most suitable for regular and symmetric algorithms. The suggested architecture exploits the regularity in fuzzy evidential reasoning systems to reduce their complexity.

A fuzzy systolic array for the *fuzzy evidential reasoning* system has been described in this paper. The architecture

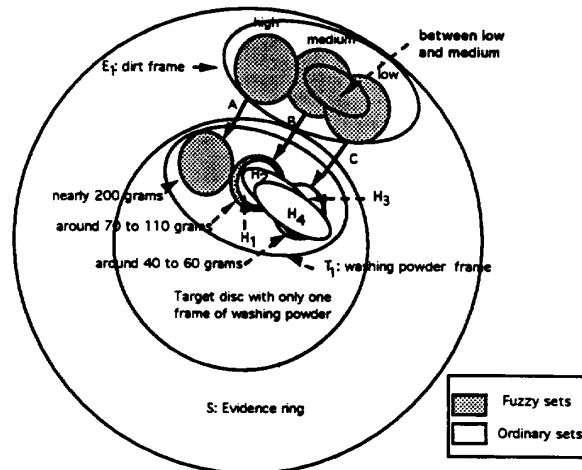


Figure 3. Evidential reasoning model for a fuzzy washing machine

is composed of mainly three modules. The first module implements the reasoning process. The Possibility and Necessity are implemented in the second module. Plausibility and Beliefs are computed in the third module by combining the Possibility and Necessity obtained from the second module. In this paper only the second and third modules of computation has been shown on systolic arrays. Design and implementation of the first module is proposed in future. For the time being in the first part all possible reasoning chains through the rules are considered to be fired. By feeding the evidence (facts) continuously to the first module, Plausibility and Beliefs are obtained continuously for different hypotheses which are fed to the second module. Therefore, this architecture is most suitable for adaptive control systems [13] which learn fuzzy rules in real-time. The architecture is linearly scalable with increase in the size of the evidence and hypothesis sets.

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