Survivable Cloud Network Mapping with Multiple Failures

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***Abstract*—Cloud computing services are realized through the mapping of the service layer network into the physical infrastructure. Multiple failures in the physical infrastructure could disrupt cloud network connectivity and cause cascading failures impacting cloud service customers. As the physical infrastructure has limited resources, most early research works for survivable virtual network mapping were concentrated on the single link failure scenario. In this paper, we study survivable cloud network mapping with multiple physical link failures and a special case, Shared Risk Link Group (SRLG) failure. We present the necessary and sufficient conditions to guarantee a survivable mapping with multiple physical link failures. Corresponding mixed-integer linear programming (MILP) formulations which avoid the enumeration of failure link combinations are proposed. We also provide the corresponding formulations for the SRLG case. Computation results demonstrate the viability of our approaches.**

Keywords*—*Cloud network, survivability, multiple failures, SRLG failures, cross-layer network, optical communication

I. INTRODUCTION

In cloud computing systems, software, hardware, physical infrastructure, service, data, and business could be considered independently or dependently as a service provided in cloud systems ("XaaS") [1]. In the cloud architecture, hardware/physical infrastructures are physical resources composed of datacenters and communication networks [2]. All cloud services above are then realized through cloud network mapping, which includes virtual-machine allocation and service layer mapping (i.e., network and service virtualization) with physical resources [3]. Hence, a reliable and stable cloud service depends on robust physical resource allocations through a dedicated mapping between service layer and physical infrastructure, which motivates the study of cloud network survivability. A cloud network is claimed to be survivable if its service sustains against failure(s) in the physical infrastructure, such as link failures in communication networks [4] or power outage in datacenters [5]. Figure 1 illustrates a cloud network, which maps a service layer network consisting of interconnected virtual machines into a physical network with datacenters and communication networks.

Optical networks as communication media connecting datacenters are used to support cloud services via optical network virtualization [6][7][8]. To protect optical networks from link failure(s), Grover and Stamatelakis [9] introduced the preconfigured cycle to achieve high capacity efficiency and fast after-failure restoration, which was further extended by Liu and

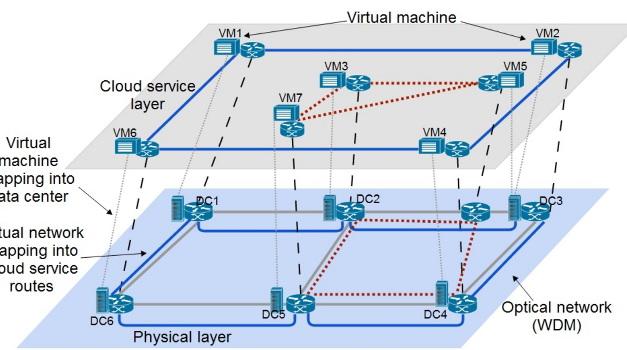

Fig. 1. Cloud network mapping

Ruan [10] against SRLG failures. Li [11] proposed the backup routing scheme by link-disjoint paths. Datta and Somani [12] discussed the diverse routings scheme for shared risk resource group.

Other than protecting the physical infrastructure mentioned above, another line of investigation to guarantee the sustainability of a cloud service is the design of survivable cross-layer networks (through virtual network mapping,) which guarantees the connectivity of the service layer network after failure(s) occur in the physical infrastructure. Due to its NP-completeness nature, early research works are mostly concentrated on a single physical link failure scenario. Kurant and Thiran [13] proposed a disjoint-path-based protection scheme to guarantee the sufficient condition for cross-layer network survivability, which was extended by Thulasiraman et al. [14]. Modiano and Narula-Tam [15] provided necessary and sufficient conditions for the existence of survivable design based on cross-layer cutsets, while Zhou et al. [16] proposed a different approach using protecting spanning trees. For multiple physical link failures, Todimala and Ramamurthy [17] studied survivable mappings with single node and single SRLG failure models. Xi et al. [18] considered the rerouting as a restoration scheme to recover SRLG failures. Similar concepts have also been applied to the survivable cloud network mapping problem. For instance, the backup node scheme in [19][20] utilized backup nodes to replace failed nodes; the re-provisioning of lightpaths approach in [21] discussed server capacity relations; and an integrated approach with content placement/replica and routing was introduced in [5]; and the anycast routing scheme in [22]. A special case, disaster survivable design, is studied in [4] by adding probability model for unpredicable disaster scenarios. We wish to note here that at least one of the sufficient conditions (disjoint-path, cross-layer cutsets, or protection spanning trees) is applied in all the above works to guarantee the survivability of a cross-layer mapping.

In this paper, we study the survivable cloud network mapping problem agsinst generalized k physical link failures, where the SRLG failure scenario is a special case. We propose necessary and sufficient conditions for survivable cloud network mapping after multiple physical link failures, and design exact solution approaches through MILP formulations. The contributions of this paper are twofold: first, to the best of our understanding, this is the first work on survivable cloud network mapping with generalized multiple link failures and we prove the necessary and sufficient conditions for this scenario. Second, we propose exact solution approaches for survivable cloud network mapping without enumerating all possible combinations of failed physical link sets.

The rest of paper is organized as follows: In Section II, we formally provide the problem description, definitions, and necessary and sufficient conditions with multiple physical link failures (including SRLG failures). In Section III, we provide and prove the MILP formulations as exact solution approaches to describe survivable mapping conditions for both SRLG failures and generalized multiple physical link failures. Finally, computational results for algorithms' performance are reported in Section IV.

II. Problem Description

TABLE I. Notations

Notation	Representation
G_P	The physical network, $G_P = (V_P, E_P)$ with V_P and E_P as physical node and edge sets
G_S	The cloud service layer network, $G_S = (V_S, E_S)$ with V_S and E_S as service layer node and edge sets
R_E	Shared risk link groups (SRLGs), whose element $r = \{(e_1^r, e_2^r, \ldots, e_k^r) : e_\ell^r \in E_P\}, r \in R_E$ represents a SRLG
T	Cloud service layer spanning tree set, $T \in G_S$ with τ as a tree
p_v	Cloud service layer link v's mapping in physical network, $p_v \subset G_P$
$\Lambda(i,j)$	A set of cloud service layer links routed through physical link (i,j), i.e., $\{(s,t) : (i,j) \in p_{st}, (s,t) \in V_S\}$

We let $G_S = (V_S, E_S), G_P = (V_P, E_P)$ denote the service layer network and physical infrastructure of a cloud network, respectively. Each $s \in V_S$ is mapped to a corresponding $u \in V_P$, while requests between nodes s and t, $s,t \in V_S$, and realized through a ***cloud network link mapping*** which is a route in G_P connecting s and t's corresponding nodes u and $v, u, v \in V_P$. Let R_E denote a SRLG set containing all SRLG failure scenarios, where each element $r \in R_E$ represents a single SRLG. Notations used in this paper are given in Table I.

A cloud service layer link mapping is called ***survivable with SRLG failure*** if its corresponding routing remains connected after any SRLG failure in R_E. If a cloud network mapping is survivable after a SRLG failure, at least a spanning tree τ should be embedded in the service layer. Next, we extend the concept of SRLG failures to multiple physical link failures in the cloud network. Given k as the total number of failed links, a cloud service layer link mapping is ***k-survivable*** if the corresponding routing remains connected after arbitrary k physical link failures.

We demonstrate the definition of survivable link mapping after SRLG failures in an example illustrated in Figure 2. The

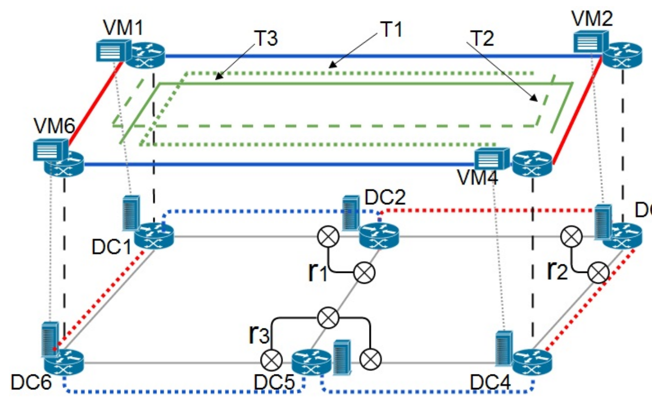

Fig. 2. Survivable cloud network mapping with SRLG failure

SRLG failure set $R_E = \{r_1, r_2, r_3\}$ with $r_1 = \{(1,2), (2,5)\}$, $r_2 = \{(2,3), (3,4)\}$, and $r_3 = \{(4,5), (2,5), (5,6)\}$. The cloud service layer link mappings are routing $p_{12} = \{(1,2)\}$, $p_{24} = \{(2,3), (3,4)\}$, $p_{46} = \{(4,5), (5,6)\}$, and $p_{16} = \{(1,6)\}$. After r_2 failure, the cloud network mapping is survivable, with the existence of cloud service layer connected spanning tree $\tau_1 = \{(1,2), (1,6), (4,6)\}$. With the existence of two other connected spanning trees $\tau_2 = \{(1,6), (2,4), (4,6)\}$ and $\tau_3 = \{(1,2), (1,6), (2,4)\}$, the cloud service layer remains connected after SRLG r_1 and r_3 failures. Hence, the given cloud network mapping is survivable after any SRLG failure in R_E.

III. Mathematical Formulations

In this section, we first present a mathematical programming formulation for the survivable cloud network mapping problem when an arbitrary set of multiple link failures occur in the physical infrastructure. We then present a formulation for the case of SRLG failures.

For a single physical link failure, there are $|V_P|$ scenarios of failure. But for k generalized physical link failures, there are scenarios $B^k_{|V_P|}$ with binomimal coefficent $B^k_n = \begin{pmatrix} n \\ k \end{pmatrix}$; for SRLG failure, there are $|R_E|$ failure scenarios, where multiple physical links fail simultaneously in the same SRLG.

Let y^{st}_{ij} be variable which represents whether (s,t)'s link mapping goes through (i,j), if yes, $y^{st}_{ij} = 1$; otherwise $y^{st}_{ij} = 0$ and $y^{st}_{i_1 j_1, \cdots, i_k j_k}$ be the variable which has value 1 if for a $(s,t) \in E_S$ the corresponding route in G_P uses one or more of the links $(i_1, j_1), \cdots, (i_k, j_k)$ where $(i_1, j_1) \neq (i_2, j_2) \neq \cdots \neq (i_k, j_k)$; otherwise, this variable equals to 0. We let μ^{ij}_{st} be a variable that equals to 0 if link $(s,t) \in E_S$ routes through $(i,j) \in E_P$; otherwise, this variable is larger than 0 and less equal to 1; and variable $\mu^{i_1 j_1, i_2 j_2, \ldots, i_k j_k}_{st}$ be the variable that is equal to 0 if the logical link (s,t) is disconnected after the failure of one or more of the links $(i_1, j_1), \ldots, (i_k, j_k)$, otherwise, the variable is greater than 0 and less equal to 1 where $(i_1, j_1) \neq (i_2, j_2) \neq \ldots \neq (i_k, j_k) \in E_P$.

For any $(i, j) \in E_P$, we let $\Lambda(i, j)$ denote the set of links $(s, t) \in E_S$ whose routes pass through (i, j). That is, for any $(i, j) \in E_P$, $\Lambda(i, j) = \{(s, t) : (i, j) \in p_{st}, (s, t) \in E_S\}$. We have the following.

Proposition 1: A cloud network route is 1-survivable if and only if for each $(i, j) \in E_P$, there exists a spanning tree $\tau \in G_S$, such that

$$\tau \cap \Lambda(i, j) = \emptyset. \tag{1}$$

This proposition follows from the fact that a mapping is survivable if and only if the service layer contains at least a spanning tree after the physical link failures.

Extending this proposition, we get the following necessary and sufficient conditions for k-survivability.

Theorem 1: A cloud network mapping is k-survivable if and only if for arbitrary k physical link failure, there exists a spanning tree $\tau \in G_S$, such that

$$\tau \cap \cup_{\beta=1}^{k} \Lambda(i_\beta, j_\beta) = \emptyset$$

where $(i_1, j_1) \neq (i_2, j_2) \neq \cdots \neq (i_k, j_k)$ and $(i_1, j_1), (i_2, j_2), \cdots, (i_k, j_k) \in E_P$.

Corollary 1: A cloud network mapping is SRLG failure survivable if and only if after any $r \in R_E$ failure, there exists a spanning tree τ, $\tau \in G_S$ and

$$\tau \bigcap \bigcup_{(i^r_{k(r)}, j^r_{k(r)}) \in r} \Lambda(i^r_{k(r)}, j^r_{k(r)}) = \emptyset. \tag{2}$$

To express the condition in Theorem 1 in terms of variable y, we first prove the following conclusion for the 2-survivability case.

Lemma 1: A service layer link $(s, t) \in \Lambda(i_1, j_1) \cup \Lambda(i_2, j_2)$, if and only if $y^{st}_{i_1 j_1, i_2 j_2} = 1$ which is implied by

$$y^{st}_{i_1 j_1, i_2 j_2} \geq y^{st}_{i_1 j_1} + y^{st}_{j_1 i_1}, \tag{3}$$
$$y^{st}_{i_1 j_1, i_2 j_2} \geq y^{st}_{i_2 j_2} + y^{st}_{j_2 i_2}, \tag{4}$$
$$y^{st}_{i_1 j_1, i_2 j_2} \leq y^{st}_{i_1 j_1} + y^{st}_{j_1 i_1} + y^{st}_{i_2 j_2} + y^{st}_{j_2 i_2}. \tag{5}$$

Proof: Proof of necessary condition: Given a service layer link (s, t) and two physical link (i_1, j_1) and (i_2, j_2), the following four cases occur, (1) $(s, t) \in \Lambda(i_1, j_1) \cap \overline{\Lambda}(i_2, j_2)$; (2) $(s, t) \in \overline{\Lambda}(i_1, j_1) \cap \Lambda(i_2, j_2)$; (3)$(s, t) \in \Lambda(i_1, j_1) \cap \Lambda(i_2, j_2)$; and (4) $(s, t) \in \overline{\Lambda}(i_1, j_1) \cap \overline{\Lambda}(i_2, j_2)$. For case 1, $y^{st}_{i_1 j_1} + y^{st}_{j_1 i_1} = 1$, so $y^{st}_{i_1 j_1, i_2 j_2} = 1$; for case 2, $y^{st}_{i_2 j_2} + y^{st}_{j_2 i_2} = 1$, then, $y^{st}_{i_1 j_1, i_2 j_2} = 1$; for case 3, $y^{st}_{i_1 j_1} + y^{st}_{j_1 i_1} = 1$ and $y^{st}_{i_2 j_2} + y^{st}_{j_2 i_2} = 1$, then, $y^{st}_{i_1 j_1, i_2 j_2} = 1$; for case 4, $y^{st}_{i_1 j_1} = y^{st}_{j_1 i_1} = 0$ and $y^{st}_{i_2 j_2} = y^{st}_{j_2 i_2} = 0$, then, $y^{st}_{i_1 j_1, i_2 j_2} = 0$. In all cases, constraints (3) to (5) hold.

Proof of sufficient condition: with constraints (3)-(5), if either $y^{st}_{i_1 j_1} + y^{st}_{j_1 i_1} = 1$ or $y^{st}_{i_2 j_2} + y^{st}_{j_2 i_2} = 1$, then, $y^{st}_{i_1 j_1, i_2 j_2} = 1$, which implies that $(s, t) \in \Lambda(i_1, j_1) \cup \Lambda(i_2, j_2)$. Meanwhile, if $y^{st}_{i_1 j_1} = y^{st}_{j_1 i_1} = y^{st}_{i_2 j_2} = y^{st}_{j_2 i_2} = 0$, then, $y^{st}_{i_1 j_1, i_2 j_2} = 0$, which implies that $(s, t) \in \overline{\Lambda}(i_1, j_1) \cap \overline{\Lambda}(i_2, j_2)$. ■

Extending Lemma 1 to the k-survivability case, we have the following.

Lemma 2: A service layer link $(s, t) \in \cup_{k=1}^{\beta} \Lambda(i_k, j_k)$, if and only if $y^{st}_{i_1 j_1, \cdots, i_k j_k} = 1$ which is implied by

$$y^{st}_{i_1 j_1, \ldots, i_k j_k} \geq y^{st}_{i_1 j_1} + y^{st}_{j_1 i_1}, \tag{6}$$
$$\ldots\ldots \tag{7}$$
$$y^{st}_{i_1 j_1, \ldots, i_k j_k} \geq y^{st}_{i_k, j_k} + y^{st}_{j_k i_k}, \tag{8}$$
$$y^{st}_{i_1 j_1, \ldots, i_k j_k} \leq \sum_{q=1}^{k} (y^{st}_{i_q j_q} + y^{st}_{j_q i_q}). \tag{9}$$

Proof of this lemma is shown in Appendix. The following properties follow from Lemma 2 and the definition of μ.

Proposition 2: The relationships between $\mu^{i_1 j_1, i_2 j_2, \ldots, i_k j_k}_{st}$ and $y^{st}_{i_1 j_1}, \ldots, y^{st}_{i_k j_k}$ are captured by the following constraints:

$$\mu^{i_1 j_1, i_2 j_2, \ldots, i_k j_k}_{st} \leq 1 - (y^{st}_{i_1 j_1} + y^{st}_{j_1 i_1}) \tag{10}$$
$$\mu^{i_1 j_1, i_2 j_2, \ldots, i_k j_k}_{st} \leq 1 - (y^{st}_{i_2 j_2} + y^{st}_{j_2 i_2}) \tag{11}$$
$$\ldots\ldots$$
$$\mu^{i_1 j_1, i_2 j_2, \ldots, i_k j_k}_{st} \leq 1 - (y^{st}_{i_k j_k} + y^{st}_{j_k i_k}) \tag{12}$$

with $(s, t) \in E_S, (i_1, j_1) \neq (i_2, j_2) \neq \cdots \neq (i_k, j_k) \in E_P$

With variable y and μ, the feasible region of y^{st}_{ij} determines a cloud network link mapping as $Y = \{y^{st}_{ij}$: Constraint (13), with $i \in V_P, (s, t) \in E_S\}$

$$\sum_{(i,j) \in E_P} y^{st}_{ij} - \sum_{(j,i) \in E_P} y^{st}_{ji} = \begin{cases} 1, & \text{if } i = s, \\ -1, & \text{if } i = t, \\ 0, & \text{if } i \neq \{s, t\}, \end{cases} \tag{13}$$

Theorem 2: A cloud network mapping is k-survivable with arbitrary k physical link failures where $k \geq 1$, if and only if the following condition are satisfied.

$$\mu^{i_1 j_1, i_2 j_2, \cdots, i_k j_k}_{st} \leq 1 - (y^{st}_{i_\beta j_\beta} + y^{st}_{j_\beta i_\beta}), \quad 1 \leq \beta \leq k \tag{14}$$

$$\sum_{(s,t) \in E_S} \mu^{i_1 j_1, \ldots, i_k j_k}_{st} - \sum_{(s,t) \in E_S} \mu^{i_1 j_1, \ldots, i_k j_k}_{ts} = \begin{cases} -1, & s = v_0, v_0 \in V_S \\ \frac{1}{|V_S| - 1}, & s \neq v_0, v_0 \in V_S \end{cases} \tag{15}$$

$$0 \leq \mu^{i_1 j_1, i_2 j_2, \ldots, i_k j_k}_{st} \leq 1, \qquad (s, t) \in E_S \tag{16}$$

Constraint (15) captures the connectivity requirements (existence of a spanning tree) through the formulation first presented in Deng and Sasaki [23].

Hence, the exact solution approach through MILP for the k-survivable cloud network mapping with multiple physical link failures is as follows:

$$\min \sum_{(s,t) \in E_S} \sum_{(i,j) \in E_P} y^{st}_{ij}$$
$$s.t. \text{ Constraints (13) to (16), (22)}$$

Given an SRLG, $r \in R_E$, we let $k(r)$ present the number of physical links in r and order links in r, i.e., $r = \{(i^r_1, j^r_1), \cdots, (i^r_{k(r)}, j^r_{k(r)})\}$.

Based on Corollary 1 and Theorem 2, we obtain the following theorem which is a special case of Theorem 2 applicable to the SRLG failure case.

Theorem 3: A cloud network mapping is survivable after SRLG failures, if and only if the following conditions are satisfied, for any $r \in R_E$,

$$\mu_{st}^{i_1^r j_1^r, i_2^r j_2^r, \ldots, i_{k(r)}^r j_{k(r)}^r} \le 1 - (y_{i_1^r j_1^r}^{st} + y_{j_1^r i_1^r}^{st}) \tag{17}$$

$$\mu_{st}^{i_1^r j_1^r, i_2^r j_2^r, \ldots, i_{k(r)}^r j_{k(r)}^r} \le 1 - (y_{i_2^r j_2^r}^{st} + y_{j_2^r i_2^r}^{st}) \tag{18}$$

$$\cdots\cdots$$

$$\mu_{st}^{i_1^r j_1^r, i_2^r j_2^r, \ldots, i_{k(r)}^r j_{k(r)}^r} \le 1 - (y_{i_{k(r)}^r j_{k(r)}^r}^{st} + y_{j_{k(r)}^r i_{k(r)}^r}^{st}) \tag{19}$$

$$\sum_{(s,t)\in E_S} \mu_{st}^{i_1^r j_1^r, \ldots, i_{k(r)}^r j_{k(r)}^r} - \sum_{(s,t)\in E_S} \mu_{ts}^{i_1^r j_1^r, \ldots, i_{k(r)}^r j_{k(r)}^r} = \begin{cases} -1, & s = v_0, v_0 \in V_S \\ \frac{1}{|V_S|-1}, & s \neq v_0, v_0 \in V_S \end{cases} \tag{20}$$

$$0 \le \mu_{st}^{i_1^r j_1^r, i_2^r j_2^r, \ldots, i_{k(r)}^r j_{k(r)}^r} \le 1, \qquad (s,t) \in E_S \tag{21}$$

where v_0 is a selected root node in the cloud service layer network.

Now we have the following MILP formulation for survivable cloud mapping under SRLG failures and minimizing the physical link utilization.

$$\min \sum_{(s,t)\in E_S} \sum_{(i,j)\in E_P} y_{ij}^{st}$$

$s.t.$ Constraints (13), (17) to (21)

$$y_{ij}^{st} \in \{0,1\}, \qquad (i,j) \in E_P, (s,t) \in E_S \tag{22}$$

IV. Computational Results

We first present our experimental design. The goal of this computational experiment is to validate and test the proposed algorithms to provide survivable cloud network mapping for both SRLG failures and generalized multiple physical link failures in dynamic cloud network environment, where each cloud service layer network could have its own node mapping and network structures.

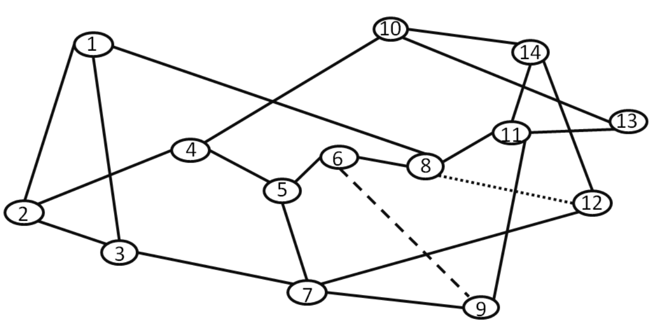

Fig. 3. NSFNet

We select NSF network as the physical infrastructure for our cloud network following [4]. The origin NSF network has 14 nodes and 21 edges illustrated with solid lines in Figure 3 and is denoted as "NSF". We let "NSF$^{(1)}$" represent the "NSF" network with augmented link $(6, 9)$, and "NSF$^{(2)}$" represent "NSF" network with augmented links $(6, 9)$ and $(8, 12)$. Two randomly generated 3-edge connected cloud service layer networks, denoted as "CLN1" and "CLN3" are illustrated in Fig. 4 in solid lines. We then augment "CLN1" and "CLN3" to 4-edge connected networks with links in dashed lines, which are denoted as "CLN2" and "CLN4" illustrated also in Figs. 4(a) and (b), respectively.

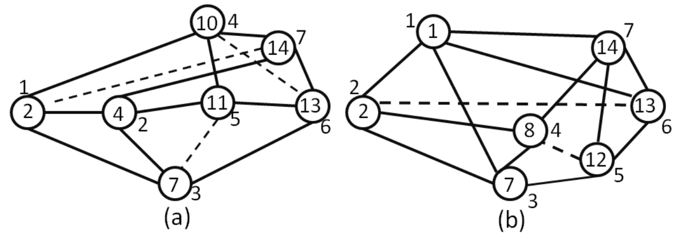

Fig. 4. Cloud service layer networks

Table II reports the information of "CLN1" to "CLN4" where "Conn", "minDeg", "maxDeg", "AvgDeg" represent the connectivity, minimal, maximal, and average node degrees of the network.

	Conn	Nodes	Edges	MinDeg	MaxDeg	AvgDeg
CLN 1	3	7	11	3	4	3.14
CLN 2	4	7	14	4	4	4
CLN 3	3	7	11	3	4	3.29
CLN 4	4	7	14	4	4	4

TABLE II. Logical topologies information

In practice, SRLG sets are known a priori. We generate SRLG failure scenarios based on 3-SRLG sets, each containing three edges. Each SRLG is not a subset of another SRLG, and its failure does not disconnect the physical network. The union of all SRLG sets covers the entire physical network [10]. For the generalized multiple physical link failure case, we consider failures of two arbitrary physical links.

We consider two evaluation metrics: (1) full survivability and maximal partial survivability; and (2) the minimal physical resource utilized to guarantee survivability/maximal partial survivability. Note here that if survivable mapping cannot be achieved due to the limitation of given networks, we report the maximal survivable design which guarantees the connectivity of cloud service layer against the most failure scenarios.

The complexity of MILP models in terms of the numbers of variables and constraints for SRLG and generalized multiple physical link failures is shown in Table III. Here $|V_P|$, $|E_P|$, $|V_S|$, $|E_S|$ represent node and edge numbers of the physical infrastructure and cloud service layer network, respectively. Let $r_{\max}$ be a SRLG set with maximum number of physical edges in R_E, and $B_n^k = \begin{pmatrix} n \\ k \end{pmatrix}$. We observe that the complexity of the formulation depends mainly on the number of failure scenarios. Given an SRLG set, the complexity of the formulation is bounded by the cardinality of the SRLG set, the number of physical links in an SRLG set, and the size of physical infrastructure and service layer network. But for the generalized k-failure problem, the complexity of the

Failure	Variables	Constraints
SRLG	$\mathcal{O}((\vert E_S\vert + \vert r_{\max}\vert)\vert R_E\vert + \vert E_P\vert)$	$\mathcal{O}((\vert V_P\vert + \vert r_{\max}\vert)\vert R_E\vert + \vert V_S\vert\vert V_P\vert)$
Generalized	$\mathcal{O}(\vert E_S\vert B^k_{\vert E_S\vert})$	$\mathcal{O}((\vert V_P\vert + k)B^k_{\vert E_S\vert}) + \vert V_S\vert\vert V_P\vert$

TABLE III. COMPLEXITY OF MILP WITH NUMBER OF VARIABLES

formulation increases exponentially with the number of failed links.

For SRLG failures, we consider NSF and NSF$^{(1)}$ as physical networks and generate a 3-SRLG set. For the constructed 3-SRLG set to cover all physical links, it requires seven 3-SRLGs, i.e., 7 elements in the 3-SRLG set, for both NSF and $NSF^{(1)}$. To generate the 3-SRLG set for $NSF^{(1)}$, the selected 3-SRLG set for NSF (denoted as R_E^{NSF}) is modified by replacing one of the overlapped edges in R_E^{NSF} with (6,9) link. We then conduct the experiments by testing our formulations with 5, 6, and 7 3-SRLGs from these 7 3-SRLGs. Let "Surv", "MaxS", "PhyS" represent the existence of survivable cloud mapping for the testing instances, maximal survivable scenarios, and minimal number of physical links in the routings. We report computational results in Tables IV and V. With NSF as the physical network, all tested cloud service networks could obtain survivable mappings with 5 and 6 3-SRLGs. But none of them could produce survivable cloud mappings with all 7 SRLGs. After augmenting NSF to NSF$^{(1)}$, survivable cloud network mappings could be achieved for all 5, 6, and 7 3-SRLGs for all tested service layer networks. Meanwhile, we observe that with both NSF and NSF$^{(1)}$ as physical networks, utilization of physical links increases when increasing the number of 3-SRLGs.

Table VI reports results for generalized 2 physical link failures. Let $SIdx$ denote the suvivability index, which shows the number of arbitrary physical link pairs whose failure does not disconnect the service layer network. We observed that with higher connectivity of the physical network, there exists survivable mapping for most testing instances. Especially for NSF$^{(2)}$, all tested service layer networks have survivable mappings. For the shortest-path based mapping, it requires some extra $[31.82\%, 76.19\%]$ of physical links. Note here that all problems are solved within 1469 seconds, and we observe that the relationship between physical link utilization and network structure is not obvious.

	5 3-SRLGs			6 3-SRLGs			7 3-SRLGs		
	Surv	MaxS	PhyS	Surv	MaxS	PhyS	Surv	MaxS	PhyS
CLN 1	Yes	5	26	Yes	6	27	No	6	38
CLN 2	Yes	5	35	Yes	6	35	No	6	42
CLN 3	Yes	5	35	Yes	6	36	No	6	39
CLN 4	Yes	5	40	Yes	6	40	No	6	44

TABLE IV. RESULTS FOR 3-SRLGS WITH NSF AS PHYSICAL NETWORK

V. CONCLUSION

In this paper, we studied the survivable cloud network mapping problem with multiple physical link failures. We proposed the necessary and sufficient conditions and corresponding MILP formulations, which can be applied as the general framework for multiple link failures in designing a survivable cross-layer network.

We have also studied the mathematical programming formulations for the SRLG case. We have presented computational results that demonstrate the viability of our approach in the design of survivable cloud network mapping for the multiple physical link failure case.

	5 3-SRLGs			6 3-SRLGs			7 3-SRLGs		
	Surv	MaxS	PhyS	Surv	MaxS	PhyS	Surv	MaxS	PhyS
CLN 1	Yes	5	30	Yes	6	30	Yes	7	37
CLN 2	Yes	5	32	Yes	6	32	Yes	7	39
CLN 3	Yes	5	34	Yes	6	50	Yes	7	48
CLN 4	Yes	5	42	Yes	6	53	Yes	7	54

TABLE V. RESULTS FOR 3-SRLGS WITH NSF$^{(1)}$ AS PHYSICAL NETWORK

	NSF			NSF$^{(1)}$			NSF$^{(2)}$		
	Surv	SIdx	PhyS	Surv	SIdx	PhyS	Surv	SIdx	PhyS
CLN 1	No	209	30	No	230	37	Yes	253	35
CLN 2	Yes	210	40	Yes	231	40	Yes	253	42
CLN 3	Yes	210	31	Yes	231	29	Yes	253	29
CLN 4	Yes	210	38	Yes	231	37	Yes	253	37

TABLE VI. RESULTS FOR GENERALIZED 2 PHYSICAL LINK FAILURES

APPENDIX A

We let $y^{st}_{i_1j_1,i_2j_2,\ldots,i_kj_k}$ indicate whether logical link (s,t) routes through $(i_1,j_1),(i_2,j_2),\ldots,(i_k,j_k)$, if yes, $y^{st}_{i_1j_1,i_2j_2,\ldots,i_kj_k} = 1$ where $u^{i_1j_1,i_2j_2,\ldots,i_kj_k}_{st} = 0$. Note here that (s,t) mapping only route through a single direction of (i,j). Then, we have following two conclusions.

Proof of Lemma 2

Proof: We prove this conclusion by induction for both necessary and sufficient conditions.
Proof of the necessary condition: with Lemma 1, if $\beta = 2$, the conclusion holds. We assume that if $\beta = k-1$, the conclusion holds. Next we show when $\beta = k$, constraints (6)-(9) still holds. With $\beta = k$, four cases occur with $[(i_1,j_1),\cdots,(i_{k-1},j_{k-1})]$ and (i_k,j_k) failures: (1) (s,t) mapping routes through (i_k,j_k) only; (2) (s,t) mapping routes through part of $[(i_1,j_1),\cdots,(i_{k-1},j_{k-1})]$; (3) (s,t) mapping routes through part of $[(i_1,j_1),\cdots,(i_{k-1},j_{k-1})]$ and (i_k,j_k); and (4) (s,t) mapping routes though none of $\{(i_q,j_q)$ with $1 \le q \le k\}$. For case 1, $y^{st}_{i_kj_k} = 1$, so $y^{st}_{i_1j_1,\cdots,i_kj_k} = 1$. For case 2, with $q = k-1$, there exists q $(1 \le q \le k-1)$, such that $y^{st}_{i_qj_q} + y^{st}_{j_qi_q} = 1$, so $y^{st}_{i_1j_1,\cdots,i_{k-1}j_{k-1}} = 1$. Hence, $y^{st}_{i_1j_1,\cdots,i_kj_k} = 1$. For case 3, if both $y^{st}_{i_kj_k} = 1$ and $y^{st}_{i_1j_1,\cdots,i_{k-1}j_{k-1}} = 1$, so $y^{st}_{i_1j_1,\cdots,i_kj_k} = 1$. These three cases imply that constraints (6)-(8) hold. For case 4, if $y^{st}_{i_1j_1} = y^{st}_{j_1i_1} = \cdots = y^{st}_{i_kj_k} = y^{st}_{j_ki_k} = 0$, then, $y^{st}_{i_1j_1,\cdots,y_kj_k} = 0$ which leads to constraint (9).
Proof of the sufficient condition: if $\beta = 2$, the conclusion holds with Lemma 1. We assume that $\beta = k-1$, the conclusion holds. Now we prove that conclusion holds when $\beta = k$, then, only two cases occur (1)$(s,t) \in \Lambda(i_k,j_k)$; and (2) $(s,t) \in \bar{\Lambda}(i_k,j_k)$. For case 1, with $y^{st}_{i_1j_1,\cdots,i_kj_k} \ge$

$y^{st}_{i_k j_k}$ which implies $(s,t) \in \cup_{\kappa=1}^{k} \Lambda(i_\kappa, j_\kappa)$. For case 2, with $\beta = k-1$, if constraints (6)-(8) leads to $(s,t) \in \cup_{\kappa=1}^{k-1} \Lambda(i_\kappa, j_\kappa)$, hence, $(s,t) \in \cup_{q=1}^{k} \Lambda(i_q, j_q)$; otherwise, $y^{st}_{i_1 j_1, \cdots, i_{k-1} j_{k-1}} \leq \sum_{q=1}^{k-1} y^{st}_{i_q j_q}$ implies $(s,t) \in \cap_{q=1}^{k-1} \overline{\Lambda}(i_q, j_q)$. With $y^{st}_{i_1 j_1, \cdots, i_{k-1} j_{k-1}} \leq \sum_{q=1}^{k-1} y^{st}_{i_q j_q} + y_{i_k j_k}$ implies $(s,t) \in \cap_{q=1}^{k} \overline{\Lambda}(i_q, j_q)$. Hence, the conclusion holds with $\beta = k$. ■

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