Logical topology augmentation for guaranteed survivability under multiple failures in IP-over-WDM optical networks

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ABSTRACT

The survivable logical topology mapping problem in an IP-over-WDM optical network is to map each link (u, v) in the logical topology (at the IP layer) into a lightpath between the nodes u and v in the physical topology (at the optical layer) such that failure of a single physical link does not cause the logical topology to become disconnected. Kurant and Thiran (2007) [8] presented an algorithmic framework called SMART that involves successive contracting of circuits in the logical topology and mapping the logical links in the circuits into edge-disjoint lightpaths in the physical topology. In a recent work from Thulasiraman et al. (2009) [11] a dual framework involving cutsets was presented and it was shown that both these frameworks possess the same algorithmic structure. Algorithms CIRCUIT-SMART, CUTSET-SMART and INCIDENCE-SMART were also presented in [11]. All these algorithms suffer from one important shortcoming, namely, disjoint lightpaths for certain groups of logical links may not exist in the physical topology. Therefore, in such cases, we will have to augment the logical topology with new logical links to guarantee survivability. In this paper we address this augmentation problem. We first identify a logical topology that admits a survivable mapping under a physical link failure as long as the physical topology is 3-edge connected. We show how to embed this logical topology on a given logical topology so that the augmented topology admits a survivability mapping as long as the physical topology is 3-edge connected. We then generalize these results to achieve augmentation for survivability of a given logical topology under multiple physical link failures. Finally, we define the concept of *survivability index* of a mapping. We provide simulation results to demonstrate that even when certain requirements of the generalized augmentation procedure are relaxed, our approach will result in mappings that achieve a high survivability index.

1. Introduction

An IP-over-WDM network implements Internet Protocol (IP) directly over a Wavelength Division Multiplexing (WDM) network by *mapping* a set of given IP connections as *lightpaths* in the WDM network [1,2]. A lightpath is an all optical connection established by finding a path between the source and the destination of an IP connection in the WDM network and assigning it a wavelength [3]. Such networks use OXCs to switch network traffic (lightpaths) in the WDM layer and IP routers to route/reroute IP connections at the IP layer [1,2]. The set of IP routers and connections form the *logical topology* and OXCs along with actual optical fibers form the *physical topology*. In the literature, it is common to refer to IP connections as IP or *logical links* (*edges*), IP routers as *logical nodes* (*vertices*), OXCs as *physical nodes* and fibers connecting the OXCs as *physical links* (*edges*).

An optical fiber simultaneously carries several lightpaths. Therefore, the failure of an optical fiber discon-

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nects all the carried lightpaths, causing multiple failures in the logical topology, which can severely impact the entire network performance. Mechanisms that allow networks to deliver an acceptable level of service in the presence of a physical edge failure or failures are referred to as *survivability mechanisms* and IP-over-WDM networks that implement such mechanisms are called *survivable IP-over-WDM networks* (henceforth, simply *survivable networks*) [2]. In this paper, we only consider link survivable networks i.e. networks that provide an acceptable level of service in the presence of one or more single link failures. The two widely discussed survivability mechanisms in the literature are *protection* and *restoration* [1,2]. Protection is generally provided at the physical layer but can be implemented at the logical layer also [1,2]. It requires a dedicated *backup lightpath* for each *working lightpath* such that the two lightpaths are link-disjoint. The backup path is used only when the working lightpath fails [2]. It is always possible to find two disjoint lightpaths, if the physical topology is at least 2-edge connected [4]. Restoration is usually provided at the logical layer by setting up working lightpaths for the IP connections and then provisioning the physical network with some additional (spare) capacity that is used by the IP routers to find backup lightpaths for the failed working lightpaths [1,2]. However, backup paths can be guaranteed only if the IP topology is initially embedded in such a way that it stays connected after a failure [5,6]. Modiano and Narula-Tam [5,6] establish the necessary and sufficient conditions for an IP-over-WDM network employing restoration to be survivable. An IP-over-WDM network employing restoration is survivable under a single link failure only if none of the cutsets of the logical topology is carried by a single physical link. However, the fact that the number of cutsets in a network is exponential in the number of nodes makes the problem intractable [7].

Kurant and Thiran [8] suggests an approach, called SMART, which finds survivable mappings for a logical-physical topology pair by successively selecting logical cycles (circuits) and finding disjoint mappings (paths) for them in the physical topology. Though the number of cycles in a logical topology grows very rapidly with the number of nodes [9], the main feature of the SMART approach is that it requires consideration of only a limited number of cycles. But, the problem of finding disjoint paths is NP-complete [10]. In [11] we established an approach that is the dual of the approach in [8] and developed a unifying algorithmic framework for the problem. We also developed several concepts and results that provided the basis for several efficient algorithms to find survivable mappings. All these algorithms suffer from one important shortcoming, namely, disjoint lightpaths for certain groups of logical links may not exist in the physical topology. Therefore, in such cases, we will have to augment the logical topology with new logical links to guarantee survivability. In this paper we address this augmentation problem.

The rest of the paper is organized as follows.

In Section 2, we present basic concepts from graph theory and review our work in [11]. We also illustrate the survivable logical topology mapping problem with an example. In Section 3 we first identify a logical topology that admits a survivable mapping under a single physical link failure as long as the physical topology is 3-edge connected. We then show how to embed this topology on a given logical topology so that the augmented topology admits a survivable mapping as long as the physical topology is 3-edge connected. In Section 4 we generalize these results to achieve augmentation for survivability of a given logical topology under multiple physical link failures. We also define the concept of the *survivability index* of a logical topology mapping. We provide simulation results to demonstrate that even when certain requirements of the augmentation procedure are relaxed, our approach will result in mappings that achieve a high survivability index. We conclude in Section 5 with a summary of our results and some directions for future work.

2. Basic concepts, survivable logical topology mapping problem and a unified algorithmic framework

The *Survivable Logical Topology Mapping* (SLTM) problem in an IP-over-WDM optical network is to map each link (u, v) in the logical topology (at the IP layer) into a lightpath between the nodes u and v in the physical topology (at the optical layer) such that failure of a physical link does not cause the logical topology to become disconnected. It is assumed that both the physical and logical topologies are 2-edge connected (in short, two-connected).

Fig. 1(a) and (b) show a logical topology and a physical topology, respectively. Fig. 1(c) shows an unsurvivable mapping of this logical topology. In this case, not all the mappings are disjoint and the logical topology is not survivable. For example, the failure of physical link $(4, 5)$ disconnects the logical topology. Fig. 1(d) shows a survivable mapping. In this case also, it can be seen that not all the mappings are disjoint and a physical link failure may disconnect multiple logical links but the logical topology still remains connected. For example, if the physical link $(5, 6)$ fails, logical links $(2, 6)$ and $(4, 6)$ get disconnected but it is possible to reach all the logical nodes through the remaining logical links. It can be observed that finding disjoint mappings for only the subset $\{(1, 2), (2, 4), (4, 6), (6, 1)\}$ is sufficient to guarantee survivability. The question then arises as to how to select the groups of logical links to be mapped into disjoint paths. The answer to this important question was provided in [8].

In [8] the authors provide a framework called SMART (Survivable Mapping Algorithm by Ring Trimming). SMART utilizes circuits to find survivable mappings for logical topologies. The framework repeatedly picks connected *pieces* (subgraphs) of the logical topology and finds survivable mappings for these pieces. If a survivable mapping is found for a piece, its links are short-circuited (contracted) and the algorithm proceeds by picking another piece. The process is repeated until the logical topology is reduced to a single node or a search for a piece with survivable mapping is unsuccessful. If the logical topology is reduced to a single node, a survivable mapping for the logical topology has been found; otherwise a survivable mapping does not exist.
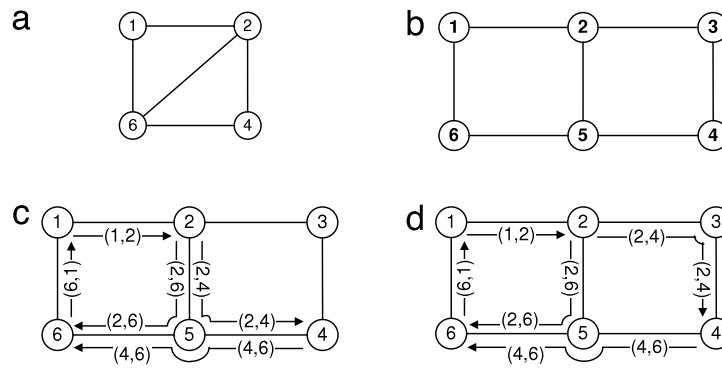
Fig. 1. Illustration of mapping and survivability for general networks: (a) A logical topology; (b) A physical topology; (c) An unsurvivable mapping; (d) A survivable mapping.

Duality between circuits and cuts in a graph is one of the well-studied topics in graph theory. This concept has played a significant role in the development of methodologies for solving problems in various applications. Most of the early results in electrical circuit theory were founded on the duality relationship between circuits and cuts [12]. There is a wealth of literature on the role of duality in network optimization (that is, discrete optimization on graphs and networks) [13]. Most often, for a primal algorithm based on circuits there is a dual algorithm based on cuts for the same problem. The primal and dual algorithms possess certain characteristics that make one superior to the other depending on the application. SMART algorithm for the survivable logical topology mapping problem is based on circuits. The question then arises whether there exists a dual methodology based on cuts. The work in [11] answered this question in the affirmative and provided a unified algorithmic framework for the SLTM problem. This work presented three methodologies CIRCUIT-SMART, CUTSET-SMART and INCIDENCE-SMART. Thulasiraman et al. [11] may be referred to for a detailed discussion of these methodologies and their proof of correctness. In this section we review INCIDENCE-SMART that will be used in the remaining sections of this paper.

Consider a connected undirected graph $G(V, E)$ with vertex set V and edge set E. G is k-edge connected if at least k edges have to be removed to disconnect G. Unless stated otherwise, all graphs considered in this section are 2-edge connected. Let $(S, \bar{S})$ be a partition of the vertex set V. Here $\bar{S}$ denotes the complement of S in V, i.e. $\bar{S} = V - S$. Then the set of edges with one vertex in S and the other in $\bar{S}$ is called a *cut* of G. This cut will also be denoted as $(S, \bar{S})$.

For example, consider the graph G in Fig. 2(a). Here the vertices are numbered as $v_1, v_2, \ldots, v_6$. The partition $(S, \bar{S})$ with $S = \{v_1, v_4, v_6\}$ and $\bar{S} = \{v_2, v_3, v_5\}$ defines the cut shown in Fig. 2(b).

The following result will be used in the proof of correctness of all the algorithms developed in the rest of the paper. This is also the basis of the algorithmic frameworks given in [11].

Theorem 1 ([12]). *A graph is connected if and only if every cut of the graph contains at least one edge.*

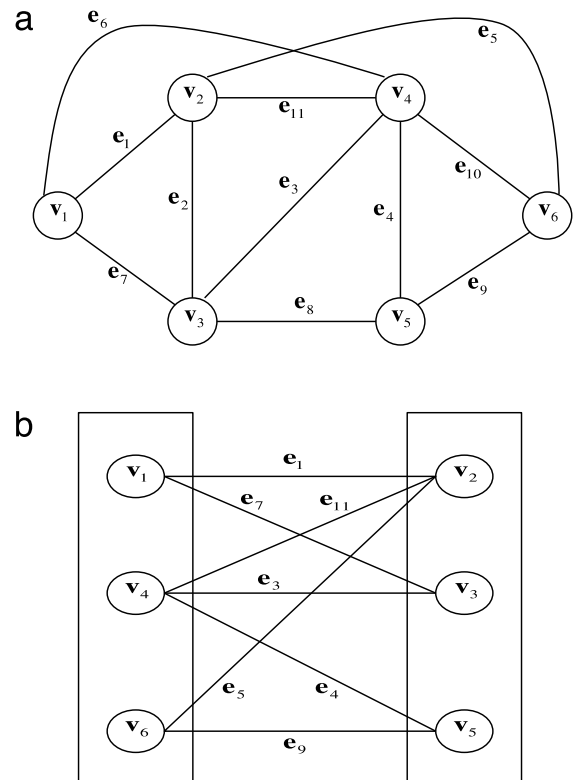We next present algorithm INCIDENCE-SMART from [11]. Given a logical topology G_L and a physical topology G_P.

Fig. 2. (a) Graph G; (b) A cut of G.

This algorithm specifies the sets of edges of G_L that must be mapped into mutually disjoint paths in the physical topology. In certain cases, the algorithm adds certain new edges to the given logical topology G_L to guarantee survivability. These new edges are added in parallel to existing logical edges and are called *protection edges*.

To illustrate algorithm INCIDENCE-SMART, consider the graph G in Fig. 2(a). It is assumed that vertex v_6 is chosen as the datum vertex. In step 1 this algorithm considers the vertices v_1, v_2, v_3, and v_4 in that order and maps the following sets of edges into mutually disjoint paths in the physical topology.

At vertex v_1 : $\{e_1, e_6, e_7\}$.

At vertex v_2 : $\{e_2, e_5, e_{11}\}$.

At vertex v_3 : $\{e_3, e_8\}$.

At vertex v_4 : $\{e_4, e_{10}\}$.

Algorithm INCIDENCE-SMART

Input: A 2-edge connected logical topology G_L and a 2-edge connected physical topology G_P. The vertices of G_L are labeled as v_1, v_2, ... v_n. Vertex v_n is called the datum vertex. Initially, G_L is the current graph.

Output: An augmented logical topology and a survivable mapping of the augmented logical topology.

BEGIN

Step 1. While there exists a vertex $v_i \neq v_n$ of degree at least 2 in the current graph, map any two of the edges incident on v_i into disjoint lightpaths in G_P and remove v_i and all the edges incident on v_i. If there is no vertex of degree 2 in the current graph, go to step 2.

Step 2. While there exists a vertex $v_i \neq v_n$ of degree one in the current graph, add a new logical edge connecting v_i to the datum vertex. Then map these two edges into disjoint lightpaths in G_P. Remove v_i. If there is no vertex of degree one, go to step 3.

Step 3. While there exists $v_i \neq v_n$ of degree zero in the current graph, add two new parallel logical edges connecting v_i to the datum vertex. Then map these two edges into disjoint lightpaths in G_P.

END

Fig. 3. Algorithm INCIDENCE-SMART.

The graphs that result after the application of step 1 on the vertices v_1, v_2, v_3, and v_4 are shown in Fig. 4(a)–(d), respectively.

In step 2, the algorithm picks vertex v_5 that is of degree 1 in Fig. 4(d), adds a new edge (v_5, v_6) (see Fig. 4(e)) and maps the parallel edges connecting v_5, v_6 into disjoint paths in the physical topology.

Proof of correctness of algorithm INCIDENCE-SMART is given below for the sake of completeness.

Theorem 2 ([11]). *Algorithm INCIDENCE-SMART provides a survivable mapping of the edges of a logical graph G_L.*

Proof. Let $v_1, v_2, \ldots, v_{n-1}$ be the order in which the vertices have been considered by algorithm INCIDENCE-SMART. Consider any cut $(S, \bar{S})$ in G_L. Let the datum vertex be in $\bar{S}$. Let v_i be the vertex in S with the highest index. Then, in the current graph at the step when v_i is considered by the algorithm it will not be adjacent to any vertex in S. So, according to the algorithm v_i will be connected to at least two vertices in $\bar{S}$, and the corresponding edges connecting S and $\bar{S}$ are mapped into disjoint lightpaths, guaranteeing that at least one of these edges will remain in the cut after a single edge failure in the physical topology and so satisfying the condition of Theorem 1. Since this is true for all cuts, the mapping generated by the algorithm is survivable. $\square$

Note that at each step, algorithm INCIDENCE-SMART requires mapping of two logical edges incident at a vertex into mutually disjoint paths in the physical topology. Such paths are guaranteed to exist as long as the physical topology is 2-edge connected [14]. We state this result in the following theorem.

Theorem 3. *If protection edges are allowed and the physical topology is 2-edge connected, then any logical topology can be augmented such that the augmented logical topology admits a survivable mapping.*

3. A survivable logical topology structure and augmentation for single link failure survivability

In this section, we first present a logical topology that always has a survivable mapping as long as the physical topology is 3-edge connected. Note that this will be achieved without using protection edges. We then show how this topology can be used to augment any logical topology to guarantee a survivable mapping of the augmented topology.

We define a graph to be k-vertex connected graph if at least k vertices have to be removed to disconnect the graph. We define the line graph of a graph as follows.

Given a graph G with m edges and n vertices, the line graph $L(G)$ of G has m vertices, with each vertex corresponding to an edge in G, and has the edge set

$\{(u, v)|$ edges in G corresponding to vertices u

   and v are adjacent$\}$.

As an example, a graph G and the line graph $L(G)$ are shown in Fig. 5.

The following result is due to Dirac [14].

Theorem 4. *Every $k \geq 2$ vertices of a k-vertex connected graph G lie on a circuit of G.*

We now prove the following. Here $P_{x,y}$ refers to the path between vertices x and y.
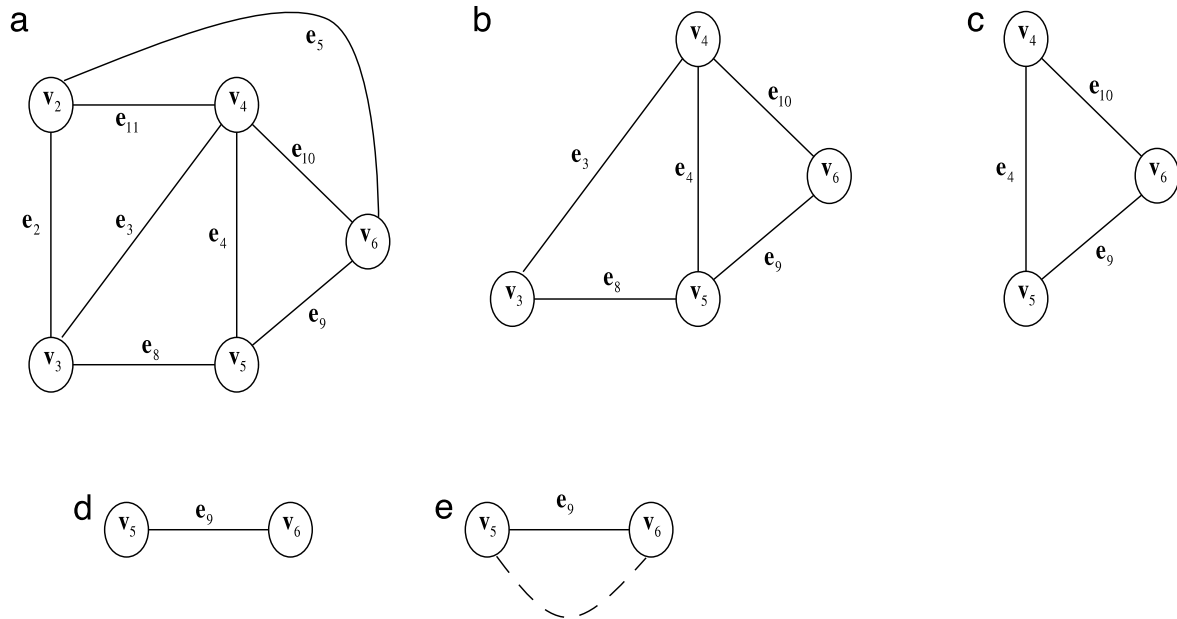
Fig. 4. (a) After removing vertex v_1 from graph G in Fig. 2(a); (b) After removing vertex v_2 from graph in Fig. 4(a); (c) After removing vertex v_3 from graph in Fig. 4(b); (d) After removing vertex v_4 from graph in Fig. 4(c); (e) New logical link (v_5, v_6) added.
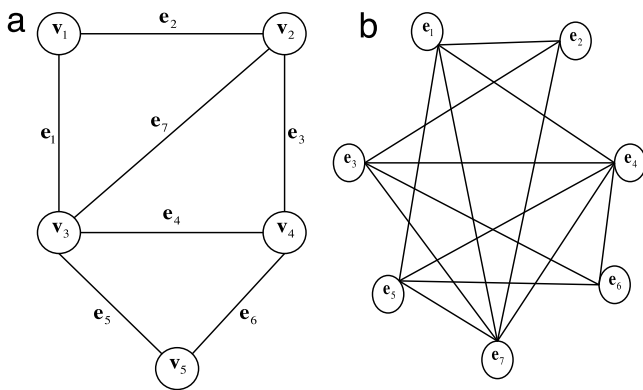

Fig. 5. (a) Graph G; (b) Line graph $L(G)$.

Theorem 5. *Given any three vertices x, y and z in a 3-edge connected graph G, then there exist edge-disjoint paths $P_{x,y}$, $P_{y,z}$ and $P_{z,x}$ in G.*

Proof. Let $G = (V, E)$ be a 3-edge-connected graph, with x, y and z in V. Form G' by adding three vertices x', y' and z', and three copies of each edge xx', yy' and zz'. By the edge analogue of the *Expansion Lemma* (adding a new vertex with three edges to old vertices), G' is 3-edge connected. The line graph $L(G')$ [14] is 3-vertex connected. By Dirac's Theorem, $L(G')$ has a shortest cycle C through vertices representing xx', yy' and zz'. Since the copies of each added edge have the same closed neighborhood in $L(G')$, this shortest cycle has only one copy each of xx', yy' and zz'. The internal vertices on the three paths joining the vertices xx', yy' and zz' on C correspond to the desired three paths in G. □

Consider next the graph $G_{n,2}$ shown in Fig. 6. This graph has n vertices $v_1, v_2, \ldots, v_n$. It has the following edges:

(1) $(v_i, v_j), i = 1, 2, \ldots, n-3$ and $j = i+1, i+2$ and
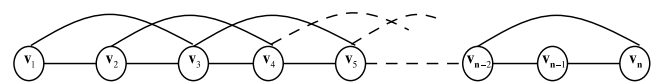(2) $(v_{n-2}, v_{n-1}), (v_{n-2}, v_n)$ and (v_{n-1}, v_n).

Fig. 6. Graph $G_{n,2}$.

Note that the three edges in (2) form a complete subgraph on the three vertices v_{n-2}, v_{n-1}, and v_n.

The mapping given in algorithm MAP-$G_{n,2}$ of Fig. 7 will be used in the proof of Theorem 6.

We now have the following result.

Theorem 6. *The logical graph $G_{n,2}$ in Fig. 6 admits a survivable mapping under a single physical edge failure if the physical topology is 3-edge connected.*

Proof. First we note that the two mutually disjoint paths required in step 1 of MAP-$G_{n,2}$ of Fig. 7 exist if the physical topology is 2-edge connected, and by Theorem 5 the three mutually disjoint paths required in step 2 of this mapping exist if the physical topology is 3-edge connected.

We now show that the mapping MAP-$G_{n,2}$ of Fig. 7 is a survivable mapping of $G_{n,2}$, thereby completing the proof of the theorem.

Case 1. Assume that v_{n-1} is not in S and let v_i be the last vertex in the sequence $v_1, v_2, \ldots, v_{n-2}$ that is in S.

In this case the vertices v_{i+1} and v_{i+2} will be in $\bar{S}$. So the edges (v_i, v_{i+1}) and (v_i, v_{i+2}) will be in the cut $(S, \bar{S})$.

Case 2. Let v_{n-1} be in S. In this case the edges (v_n, v_{n-1}) and (v_n, v_{n-2}) will be in the cut $(S, \bar{S})$.

Thus, in both cases every cut of $G_{n,2}$ will have two edges that have been mapped by MAP-$G_{n,2}$ into disjoint paths in the physical topology. So, a single physical edge failure will leave at least one edge in every cut, thereby proving (by Theorem 1) that the graph $G_{n,2}$ admits a survivable mapping under a single physical edge failure if the physical topology is 3-edge connected. □

Algorithm MAP-$G_{n,2}$

BEGIN

1. For $i = 1, 2, \ldots, n$-3, map the two edges (v_i, v_{i+1}) and (v_i, v_{i+2}) into mutually disjoint paths in the physical topology.

2. Map the edges (v_{n-2}, v_{n-1}), (v_{n-1}, v_n) and (v_{n-2}, v_n) into mutually disjoint paths in the physical topology.

END

Fig. 7. Algorithm MAP-$G_{n,2}$.

Algorithm AUGMENT (G')

BEGIN

Add additional edges to G' so that it is transformed to $G_{n',2}$ where n' is the number of vertices in G'. The original graph G along with the newly added edges is the augmented logical topology.

END

Fig. 8. Algorithm AUGMENT (G').

Given a logical topology that does not admit a survivable mapping, we next investigate how this graph can be augmented with new logical links so that the augmented graph is survivable. Our interest is to achieve this without adding protection edges. Note that there are more than one ways to construct a survivable mapping [11]. The procedure for augmentation depends on the algorithm used to construct the survivable mapping. Assuming that the algorithm INCIDENCE-SMART has been used to construct the survivable mapping. Then our procedure for augmentation will be as follows.

Note that all vertices in the graph G' at the end of the execution of step 1 in algorithm INCIDENCE-SMART will have degree zero or one. Let V' be the set of vertices in G' and E' be the set of edges in G'.

As an example, suppose the graph at the end of step 1 in algorithm INCIDENCE-SMART is as shown in Fig. 9(a). Then algorithm AUGMENT will produce the augmented graph in Fig. 9(b).

Given a logical topology G, the following algorithm AUGMENT-MAP-INCIDENCE-SMART (see Fig. 10) uses algorithm INCIDENCE-SMART (Fig. 3), algorithm AUGMENT (G') (Fig. 8) and algorithm MAP-$G_{n,2}$ (Fig. 7) to obtain an augmented topology and a mapping of the augmented topology that is survivable under a single physical edge failure, assuming that the physical topology is 3-edge connected.

Combining the proofs of Theorems 2 and 6 we obtain the following.

Theorem 7. *Given a 2-edge connected logical topology G_L and a 3-edge connected physical topology G_P, algorithm **AUGMENT-MAP-INCIDENCE-SMART** provides an augmentation of G_L and a mapping of the augmented graph that is survivable under a single edge failure in G_P.*

4. Augmentation for survivability under multiple physical edge failures

In this section we generalize the results of Section 3. First, we give a topology and a mapping that needs to be done to guarantee survivability of this topology under multiple physical edge failures. We then show how to augment a given logical topology to achieve survivability under multiple physical edge failures.

The graph $G_{n,k}$ is defined as follows. This graph has n vertices $v_1, v_2, \ldots, v_n$. It has the following edges:

1. $(v_i, v_j), i = 1, 2, \ldots, n - k - 1$ and $j = i + 1, i + 2, \ldots, i + k$ and
2. The induced subgraph on the $k+1$ vertices $v_{n-k}, v_{n-k+1}, \ldots, v_n$ is a complete graph.

As an example, the graph $G_{8,4}$ is shown in Fig. 11.

We define algorithm MAP-$G_{n,k}$, which is a generalization of MAP-$G_{n,2}$ as follows.

We now have the following result.

Theorem 8. *The logical graph $G_{n,k}$ admits a survivable mapping under $k - 1$ physical edge failures if the physical topology is k-edge connected and there exist mutually disjoint paths in the physical topology connecting the vertices of the logical edges in the complete subgraph induced on the $k + 1$ vertices $v_{n-k}, v_{n-k+1}, \ldots, v_n$.*

Proof. We first note that every cut of a complete graph on $k + 1$ vertices has at least k edges.

We prove the result by showing that every cut $(S, \bar{S})$ of $G_{n,k}$ has at least k edges that are mapped by algorithm MAP-$G_{n,k}$ into mutually disjoint paths in the physical topology. Then, MAP-$G_{n,k}$ would give a survivable mapping of G_{n-k} tolerating $k - 1$ physical edge failures.

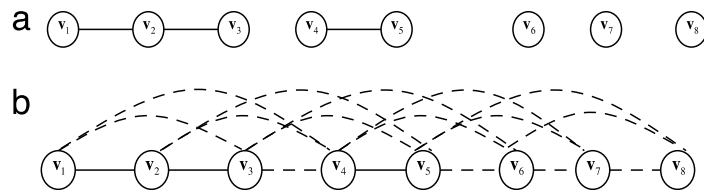Consider a cut $(S, \bar{S})$ of $G_{n,k}$. Assume node n is not in S.

Fig. 9. (a) Graph G' at the end of INCIDENCE-SMART; (b) Graph after augmentation of G'.

Algorithm AUGMENT-MAP-INCIDENCE-SMART

Input: A 2-ege connected logical topology G and a 3-edge connected physical topology G_P. Initially, G is the current graph.

Output: An augmented logical topology and a survivable mapping of the augmented topology for single physical edge failure.

Step 1. (Apply step 1 in INCIDECE-SMART) While there exists a vertex v of degree at least 2 in the current graph, map any two of the edges incident on v into disjoint lightpaths in G_P. Remove v and all the edges incident on v. If there is no vertex of degree 2 in the current graph, go to step 2.

Step 2. Apply algorithm AUGMENT(G') on the current graph G'. Label the vertices of the augmented graph as in as in Fig. 6.

Step 3. Apply algorithm MAP-$G_{n,2}$ on the augmented graph starting first from vertex v_1 in the augmented graph.

END

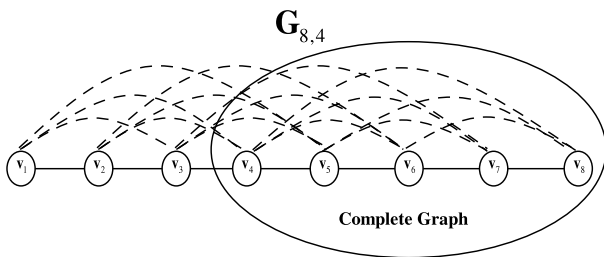**Fig. 10.** Algorithm AUGMENT-MAP-INCIDENCE-SMART.

Fig. 11. Graph $G_{8,4}$.

Case 1. All the vertices $v_{n-k}, v_{n-k+1}, \ldots, v_n$ are in $\bar{S}$.

In this case, let v_i be the last vertex from the set $\{v_1, v_2, \ldots, v_{n-k-1}\}$ that is in S. Then, by definition, v_i is adjacent to the k vertices $v_{i+1}, v_{i+2}, \ldots, v_{i+k}$ (See Fig. 11). The corresponding k edges incident on v_i are mapped by algorithm MAP-$G_{n,k}$ into mutually disjoint paths in the physical topology (See step 1 in algorithm MAP-$G_{n,k}$). Since the physical topology is k-edge connected, such mutually disjoint paths exist in the physical topology [14].

Case 2. Let T be the subset of vertices of the set $\{v_{n-k}, v_{n-k+1}, \ldots, v_{n-1}\}$ that are in S. Let $\bar{T}$ be the complement of T in $\{v_{n-k}, v_{n-k+1}, \ldots, v_{n-1}, v_n\}$. Note that these vertices are in $\bar{S}$. Then the set of edges connecting the vertices in T to those in $\bar{T}$ forms a subset of the cut $(S, \bar{S})$. This subset is in fact a cut of the complete subgraph on the $k + 1$ vertices $v_{n-k}, v_{n-k+1}, \ldots, v_n$ and has at least k edges.

Thus, we have proved that every cut $(S, \bar{S})$ of $G_{n,k}$ has at least k edges that are mapped by algorithm MAP-$G_{n,k}$

into mutually disjoint paths in the physical topology. This guarantees that every cut of $G_{n,k}$ will have at least one edge after $k - 1$ physical edge failures, thereby demonstrating that MAP-$G_{n,k}$ provides a survivable mapping of $G_{n,k}$ under $k - 1$ physical edge failures, if the conditions of the theorem are satisfied. □

We next give a generalized version of algorithm AUGMENT-MAP-INCIDENCE-SMART that achieves an augmentation of a logical topology and provides a survivable mapping of the augmented logical topology under $k - 1$ physical edge failures, provided certain requirements are satisfied. In this algorithm the augmentation procedure given in Fig. 13 is used.

Combining the proofs in Theorems 2 and 8 we obtain the following.

Theorem 9. *Given a k-edge connected logical topology G_L and a k-edge connected physical topology G_P, algorithm **GENERAL-AUGMENT-MAP-INCIDENCE-SMART** (see Fig. 14) provides an augmentation of G_L and a mapping of the augmented graph that is survivable under $k - 1$ edge failures in G_P, provided there exist mutually disjoint paths connecting the vertices of the logical edges of the complete subgraph induced on the $k + 1$ vertices $v_{n-k}, v_{n-k+1}, \ldots, v_n$.*

Algorithm MAP-$G_{n,k}$ (Fig. 12) requires that we be able to find mutually disjoint paths in the physical topology for the $k(k + 1)/2$ edges of the complete subgraph of $G_{n,k}$ on the vertices $v_{n-k}, v_{n-k+1}, \ldots, v_n$. One cannot guarantee the

Algorithm MAP-$G_{n,k}$

BEGIN

1. For $i = 1, 2, \ldots, n\text{-}k\text{-}1$,

   map the k edges (v_i, v_j), $j = i+1, i+2, \ldots, i+k$ into mutually disjoint paths in the physical topology. **Note: if the physical topology is k-edge connected, then these k mutually disjoint paths exist.**

2. Map the edges in the induced subgraph on the $k+1$ vertices $v_{n\text{-}k}$, $v_{n\text{-}k+1}, \ldots, v_n$ into mutually disjoint paths in the physical topology.

END

Fig. 12. Algorithm MAP-$G_{n,k}$.

Algorithm GENERAL-AUGMENT (G)

BEGIN

Add additional edges to G so that it is transformed to $G_{n,k}$ where n is the number of vertices in G. The original graph along with the newly added edges is the augmented logical topology.

END

Fig. 13. GENERAL-AUGMENT (G).

Algorithm GENERAL- AUGMENT-MAP-INCIDENCE-SMART

Input: A k-edge connected logical topology G and a k-edge connected physical topology $G_{P\cdot}$. Initially G is the current graph.

Output: An augmented logical topology and a survivable mapping of the augmented topology that tolerates k-1 physical edge failures.

BEGIN

Step 1. While there exists a vertex v of degree at least k in the current graph, map any k of the edges incident on v into disjoint lightpaths in G_P. Remove v and all the edges incident on v. If there exists no vertex of degree k in the current graph, go to step 2.

Step 2. Apply algorithm GENERAL–AUGMENT (G') on the current graph G'.

Step 3. Label the vertices of the augmented topology (as in Fig. 11). Apply algorithm MAP-$G_{n,k}$ on the augmented graph, starting from vertex v_1.

END

Fig. 14. Algorithm GENERAL-AUGMENT-MAP-INCIDENCE-SMART.

existence of such paths. Suppose we replace step (2) in algorithm MAP-$G_{n,k}$ by step $2'$ as follows.

   Step $2'$: For $i = n - k, n - k + 1, \ldots, n - 1$

map the edges (v_i, v_j), $\quad j = i + 1, i + 2, \ldots, n$.

Then, let us investigate how well this modified mapping is able to help tolerate $k - 1$ physical edge failures. Towards this end, we define the *survivability index* of a mapping $\prod$ with respect to a logical topology G_L as the fraction of failure patterns of a specified size under which the given logical topology remains connected when the logical links are mapped by $\prod$. We performed extensive simulations on $G_{n,k}$ for different values of n and k. In each case the physical topology is chosen as a k-connected graph generated by the procedure given in [12, chapter 8]. We considered 5000 randomly generated physical link failure patterns of size

Nodes/Failures	2	3	4	5	6
10 nodes	0.884	0.9482	0.983	0.9876	0.9938
20 nodes	0.9254	0.9604	0.9908	0.9946	0.9986
30 nodes	0.9334	0.9584	0.9932	0.9954	0.9984
40 nodes	0.9402	0.9638	0.9944	0.9946	0.999
50 nodes	0.9364	0.9732	0.9944	0.9956	0.9998
60 nodes	0.9446	0.9724	0.9952	0.9956	0.999
70 nodes	0.938	0.9768	0.996	0.9974	0.9998
80 nodes	0.937	0.9782	0.996	0.9968	0.9992
90 nodes	0.9428	0.9788	0.9948	0.9966	0.9996
100 nodes	0.9414	0.9784	0.9968	0.9982	0.999

Fig. 15. Survivability index.

$k-1$ and checked if the logical topology remains connected after the occurrence of each failure pattern. Using the number of times the logical topology tolerated the failure patterns, the survivability index is calculated in each case and is given in the table of Fig. 15.

We note that for a fixed value of k, the survivability index of the modified mapping of $G_{n,k}$ increases with n. This is because, as n increase, the number of cuts of $G_{n,k}$ that are affected by the edges involved in the modified step $2'$ decreases, thereby increasing the survivability index. For a fixed value of n, the survivability index also increases as the value of k increases. This is because, as the value of k increases, the connectivity (and hence density) of the physical topology also increases, thereby decreasing the probability of picking a failure pattern under which the logical topology gets disconnected. Overall, we find that the survivability index of the modified mapping is quite high. In other words, relaxing the requirement of step 2 in MAP-$G_{n,k}$ does not result in a significant reduction in the survivability index.

5. Summary

Given a logical topology in an IP-over-WDM optical network, we investigated the problem of augmenting this topology with additional links so that the augmented topology admits a mapping under which it remains connected when one or more physical link failures occur. We identified a special logical topology structure that can be used to achieve the required augmentation. The structure of this topology depends on the number of physical link failures that are required to be tolerated. An interesting future direction of research is to identify other structures that can be used to achieve the augmentation. In doing so, we also need to make sure that the number of additional links to be added is as small as possible.

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