Logical Topology Augmentation for Guaranteed Survivability under Multiple Failures in IP-over-WDM Optical Network

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***Abstract*—The survivable logical topology mapping problem in an IP-over-WDM optical network is to map each link (*u*, *v*) in the logical topology (at the IP layer) into a lightpath between the nodes *u* and *v* in the physical topology (at the optical layer) such that failure of a physical link does not cause the logical topology to become disconnected. Kurant and Thiran [8] presented an algorithmic framework called SMART that involves successive contracting of circuits in the logical topology and mapping the logical links in the circuits into edge disjoint lightpaths in the physical topology. In a recent work [11] a dual framework involving cutsets was presented and it was shown that both these frameworks possess the same algorithmic structure. Algorithms CIRCUIT-SMART, CUTSET-SMART and INCIDENCE-SMART were also presented in [11]. All these algorithms suffer from one important shortcoming, namely, disjoint lightpaths for certain groups of logical links may not exist in the physical topology. Therefore, in such cases, we will have to augment the logical graph with new logical links to guarantee survivability. In this paper we address this augmentation problem. We first show that if a logical topology is a chordal graph then it admits a survivable mapping as long as the physical topology is 3-edge connected and the logical topology is 2-edge connected. We identify one such chordal graph. We then show how to embed this chordal graph on a logical topology to guarantee survivability. We also show how this augmentation approach can be generalized to guarantee survivability under multiple failures.**
***Keywords*: Circuits/Cutsets duality, IP-over-WDM networks, protection, restoration, survivability.**

I. INTRODUCTION

An IP-over-WDM network implements Internet Protocol (IP) directly over a Wavelength Division Multiplexing (WDM) network by *mapping* a set of given IP connections as *lightpaths* in the WDM network [1,2]. A lightpath is an all optical connection established by finding a path between the source and the destination of an IP connection in the WDM network and assigning it a wavelength [3]. Such networks use OXCs to switch network traffic (lightpaths) in the WDM layer and IP routers to route/reroute IP connections at the IP layer [1,2]. The set of IP routers and connections form the *logical topology* and OXCs along with actual optical fibers form the *physical topology*. In the literature, it is common to refer to IP connections as *IP* or *logical links* (*edges*), IP routers as *logical nodes* (*vertices*), OXCs as *physical nodes* and fibers connecting the OXCs as *physical links*.

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An optical fiber simultaneously carries several lightpaths. Therefore, the failure of an optical fiber disconnects all the carried lightpaths, causing multiple failures in the logical topology, which can severely impact the entire network performance. Mechanisms that allow networks to deliver an acceptable level of service in the presence of a failure or failures are referred to as *survivability mechanisms* and IP-over-WDM networks that implement such mechanisms are called *survivable IP-over-WDM networks* (henceforth, simply *survivable networks*) [2]. In this paper, we only consider link survivable networks and more precisely *one link survivable networks* i.e. networks that provide an acceptable level of service in the presence of a single physical link failure.

The two widely discussed survivability mechanisms in literature are *protection* and *restoration* [1,2]. Protection is generally provided at the physical layer but can be implemented at the logical layer also [1,2]. It requires a dedicated *backup lightpath* for each *working lightpath* such that the two lightpaths are disjoint. The backup path is used only when the working lightpath fails [2]. It is always possible to find two disjoint lightpaths, if the physical topology is at least 2-edge connected [4]. Restoration is usually provided at the logical layer by setting up working lightpaths for the IP connections and then provisioning the physical network with some additional (spare) capacity that is used by the IP routers to find backup lightpaths for the failed working lightpaths [1,2]. However, backup paths can be guaranteed only if the IP topology is initially embedded in such a way that it stays connected after a failure [5,6]. [5] and [6] establish the necessary and sufficient conditions for an IP-over-WDM network employing restoration to be survivable. An IP-over-WDM network employing restoration is survivable only if none of the cutsets of the logical topology is carried by a single physical link. However, the fact that the number of cutsets in a network is exponential in the number of nodes makes the problem intractable [7].

II. A UNIFIED ALGORITHMIC FRAMEWORK BASED ON CIRCUITS/CUTSETS DUALITY

[8] suggests an approach, called SMART, which finds survivable mappings for a logical-physical topology pair by successively selecting logical cycles (circuits) and finding disjoint mappings for them in the physical topology. Since the number of cycles in a logical topology grows very rapidly with the number of nodes [9], the approach can consider a limited number of cycles only. Also, the problem of finding disjoint paths is NP-complete [10]. In [11] we established an

approach that is the dual of the approach in [8] and developed a unifying algorithmic framework for the problem. We also developed several concepts and results that provided the basis for several efficient algorithms to find survivable mappings. Some of these are discussed briefly below.

Consider a 2-edge connected undirected graph $G(V, E)$ with vertex set V and edge set E. Let T be a spanning tree of G. The edges of T are called *branches* and the remaining edges are called *chords*. Removing a branch b partitions T into two trees T_1 and T_2. The set of edges with one end in T_1 and the other in T_2 form the *fundamental cutset* ($Q(b)$) *w.r.t.* branch b. Adding a chord c to T results in exactly one circuit called *fundamental circuit* ($B(c)$) *w.r.t.* chord c. The set of all the fundamental circuits (cutsets) can be written as a matrix to form the fundamental circuit matrix B_f (fundamental cutset matrix Q_f) with respect to the spanning tree T.

An ordered sequence $B(c_1)$, $B(c_2)$, …, $B(c_k)$ is a *circuit cover sequence* or simply a *B-sequence* of length k if each $B(c_i)$, $1< i \le k$ has at least one branch that is not in any $B(c_j)$, $j <i$. The set of such branches is denoted as $S(c_i)$. The chords that are not in the circuit cover sequence are called unmapped chords. Similarly, an ordered sequence $Q(b_1)$, $Q(b_2)$, …, $Q(b_k)$ is a *cutset cover sequence* or simply a *Q-sequence* of length k if each $Q(b_i)$, $1< i \le k$ has at least one chord that is not in any $Q(b_j)$, $j < i$. The set of such branches is denoted as $\hat{S}(b_i)$. The branches that are not in the cutset cover sequence are called *unmapped branches*. The *incidence set* of a vertex v ($INC(v)$) is the set of edges incident on v. Each incident set is a cut of the graph and any set of n - 1 incidence sets can be used to generate any cut in a graph. $INC(v_1)$, $INC(v_2)$, …, $INC(v_k)$ is an *incident cover sequence* or simply an *INC-sequence* of length k if each $INC(v_i)$, $1< i \le k$ has at least one edge that is not in any $INC(v_j)$, $j < i$.

Using the above results several algorithms were proposed in [11] but only three algorithms CIRCUIT-SMART, CUTSET-SMART and INCIDENCE-SMART are discussed below. To guarantee survivability, these algorithms add new edges in parallel to some of the edges in G_L, whenever necessary. These edges will be called *protection edges*. The input to these algorithms is a physical (WDM) topology G_p and a logical (IP) topology G_L. The output of these algorithms is a survivable logical graph G'_L containing G_L.

Algorithm CIRCUIT-SMART

1. **For i =1, 2, …, k do**
   Map a maximum subset of edges in $S(c_i)\cup c_i$ into disjoint lightpaths in G_p (see [10]). To all other edges in $S(c_i)\cup c_i$ add protection edges and map each edge and its protection edge into disjoint lightpaths in G_p. **END FOR.**
2. Map all the chords not in the B-sequence into lightpaths in G_p arbitrarily. **END.**

Algorithm CUTSET-SMART

1. **For i =1, 2, …, k do**
   Map a maximum subset of edges in $\hat{S}(b_i)\cup b_i$ into disjoint lightpaths in G_p. To all other edges in $\hat{S}(b_i)\cup b_i$ add protection edges and map each edge and its protection edge into disjoint lightpaths in G_p. (see [10]).
   END FOR.
2. To each unmatched branch b add a protection edge b' and map them into disjoint lightpaths in G_p. **END.**

Algorithm INCIDENCE-SMART

1. **For i =1, 2, …, k do**
   1) If vertex v_i has degree greater than or equal to 2 in the current graph, then map any two of the edges incident on v_i into disjoint lightpaths in G_p.
   2) If the degree of v_i in the current graph is one, then add a new logical edge connecting v_i to the datum vertex. Then map this new edge and the only edge incident on v_i into disjoint lightpaths.
   3) If the degree of v_i in the current graph is zero, add two new parallel logical edges connecting v_i to the datum vertex. Then map these two edges into disjoint lightpaths in G_p.
   END FOR. END.

A simplified version of CUTSET-SMART was also presented in [11].

All of these algorithms suffer from one shortcoming, namely, for certain groups of logical links mutually disjoint paths may not exist in the physical topology. See step 1 in CIRCUIT-SMART and CUTSET-SMART. In such cases, the logical topology has to be augmented with new links to guarantee survivability.

In the following sections, we present a structure that always has a survivable mapping as long as the physical topology is 3-edge connected and the logical topology is 2-edge connected. We then show how this structure can be used to augment any logical graph to guarantee a survivable mapping. Due to space limitations, proofs of certain results will be omitted.

III. A SURVIVABLE LOGICAL TOPOLOGY STRUCTURE

The following result is due to Dirac [12].

Theorem 1: Every $k \geq 2$ vertices of a k-vertex connected graph G lie on a circuit of G. ■

We now prove the following. Here $P_{x,y}$ refers to the path between nodes x and y.

Theorem 2: Given any three vertices x, y and z in a 3-edge connected graph G, then there exist edge disjoint paths $P_{x,y}$, $P_{y,z}$ and $P_{z,x}$ in G.

Proof: Let $G= (V, E)$ be a 3-edge-connected graph, with $\{x, y, z\} \in V$. Form G' by adding three vertices x', y' and z', and three copies of each edge xx', yy' and zz'. By the edge analogue of the *Expansion Lemma* (adding a new vertex with three edges to old vertices), G' is 3-edge connected. The line graph $L(G')$ [13] is 3-vertex connected. By Dirac's Theorem, $L(G')$ has a shortest cycle C through vertices representing xx', yy' and zz'. Since the copies of each added edge have the same closed neighborhood in $L(G')$, this shortest cycle has only one copy each of xx', yy' and zz'. The internal vertices on the three paths joining the vertices xx', yy' and zz' on C correspond to the desired three paths in G. ■

A connected graph is a *chordal graph* if it has no induced circuit of length ≥ 3 [13]. A chordal graph is also referred to as a *triangulated graph*. A vertex v is a *simplicial vertex* if the induced subgraph on the neighbors of v, $N(v)$, is a complete subgraph.

Suppose the vertices of a graph are ordered as $v_1, v_2, \ldots, v_n$, then this ordering is called a *perfect elimination ordering* (PEO) if each v_i is a simplicial vertex of the induced subgraph on the vertices $v_i, v_{i+1}, \ldots, v_n$.

Proof of the following may be found in [13].

Theorem 3: A graph is chordal if and only it admits a perfect elimination ordering. ■

Theorem 4: Given a 2-edge connected logical topology G_L that is chordal, then there exists a survivable mapping of G_L if the physical topology is 3-edge connected.

Proof (*Hint*): A perfect elimination ordering of a chordal graph can be used to construct an incidence cover. Apply step 1 of algorithm INCIDENCE-SMART on the first n - 3 vertices and then map the three links on the last three vertices into edge disjoint paths in the physical topology. Such a mapping is guaranteed to exist by Theorem 2 if the physical topology is 3-edge connected. ■

A chordal graph is shown in Fig.1. The vertices v_1, v_2, …, v_n define a PEO.

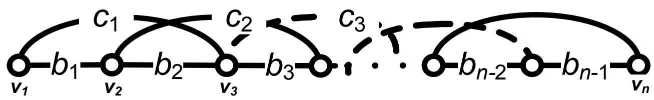

Figure 1. A chordal graph.

Using a generalized version of Theorem 2, we can generalize Theorem 4 to construct logical topologies that admit survivable mappings under multiple failures.

Details of this construction are omitted due to space limitations.

IV. LOGICAL TOPOLOGY AUGMENTATION FOR GUARANTEED SURVIVABILITY

Given a logical topology that does not admit a survivable mapping, we next consider how this graph can be augmented with new logical links so that the augmented graph is survivable. First we need to identify the bottleneck edges that are the cause of the problem and them add appropriate logical links. We believe that there is more than one way of augmenting. The following procedure shows how to augment if the set of bottleneck edges forms a tree. It can be proved that the newly added logical links along with the original graph will admit a survivable mapping if the physical topology is 3-edge connected. The proof of this result is based on certain features of the CIRCUIT-SMART, CUTSET-SMART and INCIDENCE-SMART algorithms.

Procedure AUGMENT:

1. Set $T' = T$
2. Select a path between any two leaves in T'. Augment this with new logical links as given by the structure in Fig.1. If the path has only one edge then add a parallel edge to augment this edge
3. Let T' be the new graph that results after removing the edges from the selected path.
4. Repeat step 1 on the current T' if it is not empty. Otherwise, **END.**

Suppose the bottleneck edges form trees T_1, T_2, …, T_p. Then for each tree T_i, we can add new logical links as in the above procedure. We can then show that the original logical topology augmented with the new logical links will admit a survivable mapping as long as the physical topology is 3-edge connected. Note that this procedure can be used in step 1 of CIRCUIT-SMART and steps 1 and 2 of CUTSET-SMART.

V. CONCLUSION

Survivable logical topology mapping algorithms presented in section II suffer from one important shortcoming, namely, disjoint lightpaths for certain groups of logical links may not exist in the physical topology. Therefore, in such cases, we will have to augment the logical graph with new logical links to guarantee survivability. In this paper, we considered this problem, called the logical topology augmentation problem. We have proved chordal graphs admit survivable mappings if the physical topology is 3-edge connected. We have also presented a procedure for augmenting a logical topology that is not survivable so that the augmented topology admits a survivable mapping. Using a generalized version of Theorem 2, we can generalize Theorem 4 to construct logical topologies that admit survivable mappings under multiple failures. Further research on this problem is in progress along the following lines: Since the logical topology mapping problem involves two graphs, a logical graph that is survivable with respect to one physical topology may not admit such a mapping with respect to a different physical topology. So we are investigating the characteristics of physical topologies that possess disjoint paths between a given pair of nodes in the logical topology.

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