JOURNAL OF INDUSTRIAL AND
MANAGEMENT OPTIMIZATION
Volume **3**, Number **4**, November **2007**

Website: http://AIMsciences.org

pp. **625–643**

ON THE PARTIAL PATH PROTECTION SCHEME FOR WDM OPTICAL NETWORKS AND POLYNOMIAL TIME COMPUTABILITY OF PRIMARY AND SECONDARY PATHS

Guoliang Xue, Weiyi Zhang, Tie Wang
Department of Computer Science and Engineering
Arizona State University, Tempe, AZ 85287, USA

Krishnaiyan Thulasiraman
School of Computer Science
University of Oklahoma, Norman, OK 73019, USA

(Communicated by Yiju Wang)

Abstract. As a generalization of the traditional path protection (PP) scheme in WDM networks where a backup path is needed for each active path, the partial path protection (PPP) scheme uses a collection of backup paths to protect an active path, where each backup path in the collection protects one or more links on the active path such that every link on the active path is protected by one of the backup paths. While there is no known polynomial time algorithm for computing an active path and a corresponding backup path using the PP scheme for a given source destination node pair, we show that an active path and a corresponding collection of backup paths using the PPP scheme can be computed in polynomial time, whenever they exist, under each of the following four network models: (a) dedicated protection in WDM networks without wavelength converters; (b) shared protection in WDM networks without wavelength converters; (c) dedicated protection in WDM networks with wavelength converters; and (d) shared protection in WDM networks with wavelength converters. This is achieved by proving that that for any given source s and destination d in the network, if one candidate active path connecting s and d is protectable using PPP, then any candidate active path connecting s and d is also protectable using PPP. It is known that the existence of PP implies the existence of PPP while the reverse is not true. We demonstrate a similar result in the case of segmented path protection. This fundamental property of the PPP scheme is of great importance in the context of achieving further research advances in the area of protection and restoration of WDM networks.

1. **Introduction.** All-optical networks employing wavelength division multiplexing (WDM) and wavelength routing are candidates for future high speed backbone networks [4, 9, 19]. To support mission-critical connection requests, a number of protection schemes for WDM networks have been proposed [1, 6, 10, 11, 13, 14, 16, 17, 34, 35, 36]. Among these schemes, path protection (PP) and link protection (LP)

2000 *Mathematics Subject Classification.* Primary: 68M10, 68M15; Secondary: 68Q25, 68W40.

Key words and phrases. Network design and communication, WDM networks, backup multiplexing, partial path protection, polynomial time algorithms.

The research of Xue, Zhang, and Wang was supported in part by NSF ITR grant ANI-0312635 and ARO grant W911NF-04-1-0385. The research of Thulasiraman was supported in part by NSF ITR grant ANI-0312435.

have attracted the most attention [1, 8, 16, 17, 21, 35]. PP is achieved by reserving a backup path which is link-disjoint with the active path so that the traffic on the active path can be rerouted through the backup path when a link on the active path fails. LP is achieved by reserving a backup path for each wavelength channel on the active path. When a link fails, the traffic through a wavelength channel on that link will be rerouted using its corresponding backup path. A channel on an active path cannot be used by another active path or backup path. In dedicated path/link protection, a channel on a backup path cannot be used by another backup path. In shared path/link protection, a channel on a backup path can be used by another backup path as long as the failure of any link does not activate both backup paths. In general, PP is more capacity efficient than LP [21].

In an interesting paper [18], Mohan *et al.* introduce the concept of *primary-backup multiplexing* which generalizes shared path protection. In that scheme a primary lightpath and one or more backup paths may share a channel. This approach is useful in the case of dynamic traffic where the lightpaths are short-lived. It increases channel utilization and improves network blocking performance. However, if this scheme is used a primary lightpath is not guaranteed to have a backup path readily available throughout its existence. Another idea of interest to us in the context of our paper is the concept of *segmented path protection* studied in [12, 25, 32, 33]. In segmented path protection, a primary path is broken into segments and each segment is provided a protection segment. Gummadi *et al.* [12] provide an algorithm for routing with segmented backup paths achieving optimality with respect to spare resource reservation. On the other hand Xu *et al.* [33] present an *integer linear programming* (ILP) algorithm and a heuristic method based on dynamic programming to determine an optimal/near-optimal set of segments to protect a primary active path.

In another paper [30], Wang, Modiano and Médard introduce the concept of *partial path protection* (PPP). The idea of PPP is to use a collection of one or more backup paths for each active path, so that the collection of backup paths *collectively* protect all channels on the active path. They demonstrate that **PPP is more powerful than PP in the sense that the existence of PP implies the existence of PPP while the reverse is not true**. They consider a dynamic call-by-call system with random arrivals of connection requests and present an ILP formulation to compute an active path and its corresponding PPP with minimum total cost. They also present a *shortest active path first* (SAPF) heuristic for computing an active path and its corresponding PPP with low total cost. Simulation results demonstrate that the SAPF heuristic has very good performance.

In the PP scheme, the backup path assigned to a primary path does not depend on the link that has failed. On the other hand, in the case of other path based protection schemes the backup path selected depends on the link that has failed. These latter schemes, including the PPP scheme, may therefore be viewed as failure dependent protection schemes. In this context we wish to draw attention to two recent failure dependent schemes. In [7], Frederick *et al.* introduce the concept of *subgraph routing.* Here a network is mapped to $|E|$ distinct subgraphs that are obtained by removing one link at a time from the original network, where E is the set of links in the network. A connection request is accepted only if it is accepted in all subgraphs. Thus whenever a link fails, the network can take on the state of the corresponding subgraph, possibly rerouting all of the connections in the network to recover from the fault. Though this scheme is shown to have good network utilization, Ramasubamaniamn [22] points out that the subgraph routing scheme may result in a large number of reconfigurations that could significantly affect the reconfiguration time. Ramasubramaniam also presents a *failure dependent* scheme based on the MICRON framework, an earlier work by the author [26]. In this scheme a connection on a primary path is rerouted on its backup path only if a failure in the network affects the primary path. It is demonstrated that this scheme results in a significantly large reduction in the number of reconfigurations required with minimal reduction in network utilization. It is also pointed out in [22] that this scheme is applicable to any general failure scenario that could be modeled as SRLG failures.

We wish to note that there are also other works that have studied failure dependent schemes. Such schemes in the context of ATM networks may be found in [31] and the references therein. Some related works in the context of optical networks may be found in [15, 23, 24, 28, 29].

In this paper, we prove a fundamental property of PPP. In particular, we prove that **if partial path protection exists for one candidate active path, then partial path protection exists for any candidate active path**. An immediate implication of this property is that we can always use the shortest active path while using PPP. This justifies the use of the SAPF heuristic presented by Wang, Modiano and Médard in [30]. We also present polynomial time algorithms for computing an active path and its corresponding PPP, whenever they exist. If our algorithm fails to find an active path and its corresponding PPP for a given connection request, then the connection is not protectable under the PPP scheme and hence will be rejected. Note that computing an active path and its corresponding backup path connecting a source-destination node pair using the dedicated path protection scheme in a WDM network without wavelength converters has been shown to be NP-complete by Andersen, Chung, Sen and Xue [2]. In a more recent paper, Ou *et al.* [20] prove that computing an active path and its corresponding backup path using shared path protection is also NP-complete. Therefore our polynomial time algorithms demonstrate an important advantage of PPP over PP.

The rest of the paper is organized as follows. In Section 2, we present some basic definitions about WDM networks and the protection schemes PP and PPP that will be used in subsequent sections. In Section 3, we present a fundamental property of dedicated partial path protection in a WDM network without wavelength converters and a polynomial time algorithm for computing an active lightpath and its dedicated partial path protection, whenever they exist. We also state similar properties without proofs and present algorithms for three other models: shared partial path protection in a WDM network without wavelength converters, dedicated partial path protection in a WDM network with wavelength converters and shared partial path protection in a WDM network with wavelength converters. In Section 4, we discuss the relationship between the partial path protection scheme and the segmented path protection scheme. Specifically, we show that **the existence of segmented path protection implies the existence of PPP while the reverse is not true**. In Section 5, we summarize our results and also point out the significance of our work in the context of polynomial time computability of different path (including segmented path) based protection schemes, and the freedom and flexibility the partial path protection scheme affords in accommodating QoS guarantees while selecting both the active path and the corresponding set of backup paths.

2. **Basic Definitions.**

We model a WDM network using an undirected graph $G = (V, E, \Lambda)$, where V is the set of n *vertices*, denoting the *nodes* in the network; E is the set of m *edges*, denoting the *links* (or optical fibers) in the network; $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_W\}$ is the set of W *wavelengths* each link is capable of carrying. We will use the terms vertices and nodes interchangeably, as well as edges and links. We will use *channel* to denote a wavelength on a particular link. Specifically, we will use e^λ to denote the channel which uses wavelength λ on link e. For any link $e \in E$, $\Lambda^A(e) \subseteq \Lambda$ denotes the set of wavelengths (called *active channels*) on link e that are *used by active paths* of existing connections; $\Lambda^R(e) \subseteq \Lambda \setminus \Lambda^A(e)$ denotes the set of wavelengths (called *reserved channels*) on link e that are *used by backup paths* of existing connections; $\Lambda^F(e) \subseteq \Lambda \setminus \{\Lambda^A(e) \cup \Lambda^R(e)\}$ denotes the set of wavelengths (called *free channels*) on link e that are not used by either active paths or backup paths of existing connections.

In a WDM network without wavelength converters, data transmission is carried out on a lightpath. Following Chlamtac *et al.* [4], A *lightpath* $\pi^\lambda(s, d)$ between nodes $s, d \in V$ on wavelength $\lambda \in \Lambda$ is an s–d path $\pi(s, d)$ in G which uses wavelength λ on every link of path $\pi(s, d)$. $\pi(s, d)$ is called the *basepath* of lightpath $\pi^\lambda(s, d)$. λ is called the *wavelength* of lightpath $\pi^\lambda(s, d)$. Note that all channels on a lightpath must be on the same wavelength. This is known as the *wavelength continuity constraint.*

In a WDM network with wavelength converters, the wavelength continuity constraint is no longer required. Following Chlamtac *et al.* [3], A *semi-lightpath* $\pi^{sem}(s, d)$ between nodes $s, d \in V$ is a sequence of interleaving nodes and wavelengths $s = u_0$, $\lambda(u_0, u_1)$, u_1, $\lambda(u_1, u_2)$, …, $\lambda(u_{k-1}, u_k)$, $u_k = d$ such that $(u_0, u_1, \ldots, u_k)$ form an s–d path (the basepath) and $\lambda(u_{j-1}, u_j) \in \Lambda$ for $j = 1, 2, \ldots, k$.

To protect a mission-critical connection from any single link failure, we need to set up an *active path* and its corresponding *backup* to protect against the failure of a link along the active path. Since each link has many wavelengths, several lightpaths or semi-lightpaths may pass through the same link. As a result, the failure of a particular link e may affect all lightpaths or semi-lightpaths which go through link e. A direct consequence of the above observation is that *the backup path should not use any of the links it is protecting.* This constraint is enforced in all commonly known protection schemes.

In PP [21], for every connection request ρ with source node $s(\rho)$ and destination node $d(\rho)$, we need to establish an active path $\mathcal{AP}(\rho)$ connecting $s(\rho)$ and $d(\rho)$. We also need to establish a backup path $\mathcal{BP}(\rho)$ connecting $s(\rho)$ and $d(\rho)$ which is link-disjoint with $\mathcal{AP}(\rho)$. PP could be either *shared* or *dedicated.* In shared PP, the backup paths of two active paths may share a channel if and only if the two active paths do not share a link. In dedicated PP, no two backup paths can share a channel. Fig. 1(a) illustrates an active path A_1: *b-a-u-w-y* on wavelength λ_1 and an active path A_2: *b-v-x-z* on wavelength λ_1, as well as their backup paths *using shared path protection.* A_1 is protected by the backup path B_1: *b-v-x-z-y* on wavelength λ_2. A_2 is protected by the backup path $B2$: *b-u-w-y-z* on wavelength λ_2. Fig. 1(b) illustrates an active path A_1: *b-a-u-w-y* on wavelength λ_2 and an active path A_2: *b-v-x-z* on wavelength λ_2, as well as their backup paths *using dedicated path protection.* A_1 is protected by the backup path B_1: *b-v-x-z-y* on wavelength λ_1. A_2 is protected by the backup path B_2: *b-a-u-w-z* on wavelength λ_1.

In PPP [30], for every connection request ρ with source node $s(\rho)$ and destination node $d(\rho)$, we need to establish an active path $\mathcal{AP}(\rho)$ connecting $s(\rho)$ and $d(\rho)$. We

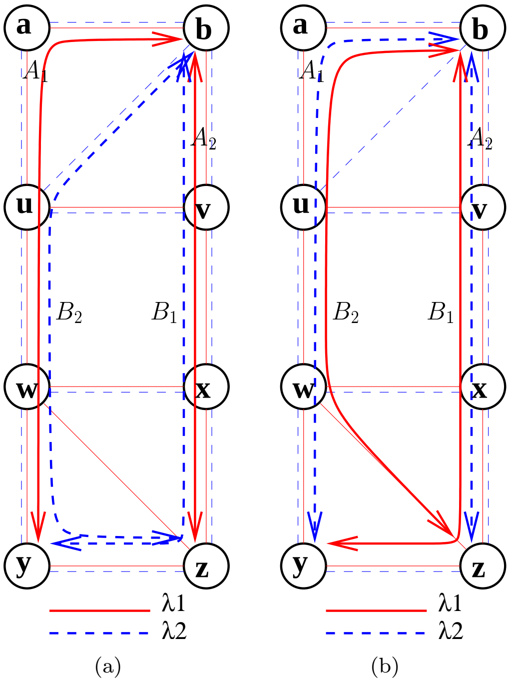
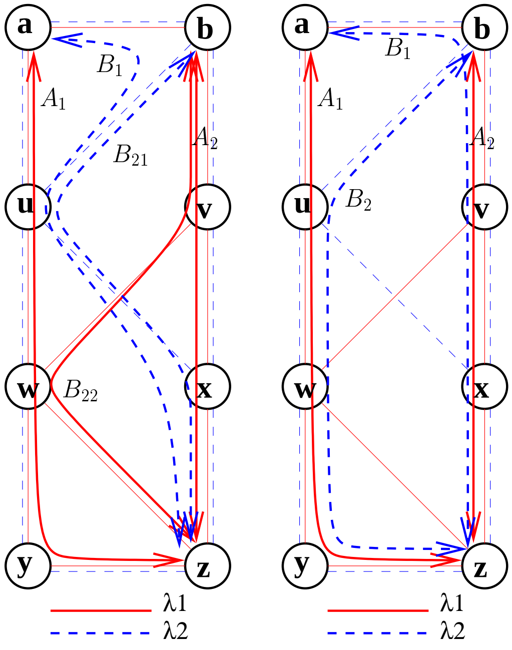


FIGURE 1. (a) Shared path protection: A_1 is an active path on λ_1, A_2 is an active path on λ_1, B_1 is the backup path for A_1, B_2 is the backup path for A_2. (b) Dedicated path protection: A_1 is an active path on λ_2, A_2 is an active path on λ_2, B_1 is the backup path for A_1, B_2 is the backup path for A_2.

also need to establish *a collection of one or more backup paths* $\mathcal{BP}(\rho)$ each connecting $s(\rho)$ and $d(\rho)$ such that for every link e on $\mathcal{AP}(\rho)$, there is a corresponding backup path $\mathcal{BP}(\rho, e) \in \mathcal{BP}(\rho)$ which does not use link e, but may share links and/or channels with the rest of $\mathcal{AP}(\rho)$. Note that we may have $\mathcal{BP}(\rho, e_1) = \mathcal{BP}(\rho, e_2)$ for two different links e_1 and e_2 on $\mathcal{AP}(\rho)$. Note also that we are talking about a backup path for a channel on the active path. Partial path protection is different from path protection where the backup path protects the *entire active path*, rather than part of the active path. Again, partial path protection could be either *shared* or *dedicated*. In shared partial path protection, the backup path $\mathcal{BP}(\rho, e)$ of one active path $\mathcal{AP}(\rho)$ may share a channel with the backup path $\mathcal{BP}(\sigma, f)$ of another active path $\mathcal{AP}(\sigma)$ if and only if the links on $\mathcal{AP}(\rho)$ that $\mathcal{BP}(\rho, e)$ is supposed to protect do not intersect the links on $\mathcal{AP}(\sigma)$ that $\mathcal{BP}(\sigma, f)$ is supposed to protect. In dedicated partial path protection, the backup path $\mathcal{BP}(\rho, e)$ of one active path $\mathcal{AP}(\rho)$ cannot share a channel with the backup path $\mathcal{BP}(\sigma, f)$ of another active path $\mathcal{AP}(\sigma)$. However, two backup paths for the same active path may share channels. Fig. 2 illustrates both shared and dedicated partial path protections.

Fig. 2(a) illustrates two active paths A_1 and A_2 and their corresponding (*shared*) partial path protections. For A_1, a single path B_1 protects all links on A_1. For A_2, we have two backup paths. B_{21} is used to protect links b-v and v-x on A_2, B_{22} is used to protect link x-z on A_2. We note that B_1 and B_{21} share several channels. We



(a) (b)

FIGURE 2. (a) Shared partial path protection: A_1 (a-u-w-y-z) is an active path on λ_1, A_2 (b-v-x-z) is an active path on λ_1, B_1 (a-b-u-x-z on λ_2) is the backup path for all links on $A1$, B_{21} (b-u-x-z on λ_2) is the backup path for links b-v and v-x on A_2, B_{22} (b-v-w-z on λ_1) is the backup path for link x-z on A_2. (b) Dedicated partial path protection: A_1 is an active path on λ_1, A_2 is an active path on λ_1, B_1 is the backup path for all links on A_1, B_2 is the backup path for all links on A_2.

also note that B_{22} shares channel b-v (on λ_1) with active path A_2. When link x-z fails, the traffic between v and z on A_2 will be rerouted via the path v-w-z on λ_1. Fig. 2(b) illustrates two active paths A_1 and A_2 and their corresponding (*dedicated*) partial path protections. For A_1, a single path B_1 protects all links on A_1. For A_2, a single path B_2 protects all links on A_2. Note that if we fix the protection for A_1 as in Fig. 2(a), we would not be able to find dedicated partial path protections for A_2.

Wang, Modiano and Médard [30] have shown that for any given connection request, the existence of an active path and its corresponding path protection implies the existence of an active path and its corresponding partial path protection, but the reverse is not true. Therefore partial path protection is a very promising protection scheme. In this paper, we study partial path protection under four different network models and prove a fundamental property of the PPP scheme. As in [30], we consider a dynamic call-by-call system where connection requests arrive sequentially. For each connection request, we will block it only if it is impossible to establish an active path and its corresponding partial path protections.

Let $e \in E$ be a link in the network. We use $\mathcal{AC}(e)$ to denote the set of connections whose active lightpaths pass through link e. We use $\mathcal{BC}(e)$ to denote the set of connections whose backup lightpaths pass through link e. We will use *existing active path* to mean an active path of an existing connection. We will use *existing backup path* to mean the backup path for some links on an existing active path. We will use the term *active path* to mean a candidate for the active path of the connection request under consideration.

3. Polynomial Time Computation of Primary and Secondary Paths under PPP.

In this section, we first consider *dedicated partial path protection in a WDM network without wavelength converters.* Since there are no wavelength converters in the network, each active path or backup path must be a lightpath. For this reason, we use the term *lightpath connection.* We then consider partial path protection under three other models. Our remarks in the case for the dedicated partial path protection scheme without wavelength converters are also applicable for other models. The notation $\mathcal{AP}$ ($\mathcal{BP}$) used in Section 2 will be changed to $\mathcal{AL}$ ($\mathcal{BL}$) for lightpath routing and changed to $\mathcal{ASL}$ ($\mathcal{BSL}$) for semilightpath routing in this section.

Definition 3.1. [**Lightpath Connection with Dedicated Partial Path Protection (LPDPPP)**] Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. A lightpath connection with dedicated partial path protection for ρ consists of an active path $\mathcal{AL}(\rho)$ and a set of backup paths $\mathcal{BL}(\rho)$ corresponding to $\mathcal{AL}(\rho)$, where $\mathcal{AL}(\rho)$ is a lightpath connecting $s(\rho)$ and $d(\rho)$, $\mathcal{BL}(\rho)$ is a set of lightpaths each connecting $s(\rho)$ and $d(\rho)$ such that the following conditions are satisfied:

A1: The lightpath $\mathcal{AL}(\rho)$ uses free wavelength channels only.
A2: For each link e on $\mathcal{AL}(\rho)$, there is a corresponding lightpath $\mathcal{BL}(\rho, e) \in \mathcal{BL}(\rho)$ such that $\mathcal{BL}(\rho, e)$ does not use link e. $\mathcal{BL}(\rho, e)$ is the backup path of link e on $\mathcal{AL}(\rho)$. $\mathcal{BL}(\rho, e)$ may share channels with $\mathcal{AL}(\rho)$. Also, $\mathcal{BL}(\rho, e_1)$ may share channels with $\mathcal{BL}(\rho, e_2)$ for two different links e_1 and e_2 on $\mathcal{A}(\rho)$.
A3: Every lightpath in $\mathcal{BL}(\rho)$ uses only free wavelength channels.

Let $\mathcal{AL}(\rho)$ be an $s(\rho)$–$d(\rho)$ lightpath using only free wavelength channels. We say that lightpath $\mathcal{AL}(\rho)$ is dedicated partial path protectable if there exists a set of backup paths $\mathcal{BL}(\rho)$ such that conditions A1–A3 are satisfied. In this case, we say that $\mathcal{BL}(\rho)$ is the dedicated partial path protection of active lightpath $\mathcal{AL}(\rho)$.

One can immediately notice the following difference between the traditional path protection scheme and the partial path protection scheme. In path protection, a single backup path is used to protect all links on the corresponding active path. In partial path protection, all links on the active path are protected, but two different links on the active path may be protected using two different backup paths. This fundamental difference leads to an important property of the partial path protection scheme that is discussed below.

Given a candidate active lightpath connecting the source node and the destination node, the existence of a link-disjoint backup lightpath can be decided efficiently. However, it may happen that for one candidate active lightpath there is a link-disjoint backup lightpath, but for another candidate active lightpath there is no link-disjoint backup lightpath. This shows that **choosing the correct active lightpath is critical to the existence of a link-disjoint dedicated backup**

lightpath. Since the number of possible active lightpaths between a given source-destination node pair may be exponential, this reveals the difficulty of the path protection scheme. Actually, Andersen *et al.* [2] has shown that computing a pair of link-disjoint lightpaths connecting $s(\rho)$ and $d(\rho)$ is an NP-hard problem.

In the following, we will show that **if one active lightpath connecting a given source-destination node pair is dedicated partial path protectable, then any active lightpath connecting the same source-destination node pair is also dedicated partial path protectable**. We will then use this fundamental property to design an efficient algorithm for establishing a lightpath connection with dedicated partial path protection. This fact makes the partial path protection scheme more attractive than the traditional path protection scheme.

Theorem 3.2. *Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. Let $\mathcal{AL}_1(\rho)$ and $\mathcal{AL}_2(\rho)$ be two $s(\rho)$–$d(\rho)$ lightpaths using only free wavelength channels. If there exists a set of lightpaths $\mathcal{BL}_1(\rho)$ so that $\mathcal{AL}_1(\rho)$ and $\mathcal{BL}_1(\rho)$ form a lightpath connection with dedicated partial path connection for ρ with $\mathcal{AL}_1(\rho)$ as the active path, then there exists a set of lightpaths $\mathcal{BL}_2(\rho)$ so that $\mathcal{AL}_2(\rho)$ and $\mathcal{BL}_2(\rho)$ form a lightpath connection with dedicated partial path connection for ρ with $\mathcal{AL}_2(\rho)$ as the active path. In other words, $\mathcal{AL}_1(\rho)$ is dedicated partial path protectable if and only if $\mathcal{AL}_2(\rho)$ is dedicated partial path protectable.*

Proof. We define $\mathcal{BL}_2(\rho)$ to be the set $\{\mathcal{BL}_2(\rho, e) | e \in \mathcal{AL}_2(\rho)\}$ with $\mathcal{BL}_2(\rho, e)$ defined in the following.

Let e be any link on $\mathcal{AL}_2(\rho)$. If e is not on $\mathcal{AL}_1(\rho)$, we define $\mathcal{BL}_2(\rho, e) = \mathcal{AL}_1(\rho)$. If e is on $\mathcal{AL}_1(\rho)$, we define $\mathcal{BL}_2(\rho, e) = \mathcal{BL}_1(\rho, e)$. We need to show that $\mathcal{AL}_2(\rho)$ and $\mathcal{BL}_2(\rho)$ satisfy conditions A1-A3 in Definition 3.1, i.e., $\mathcal{BL}_2(\rho)$ is a dedicated partial path protection for $\mathcal{AL}_2(\rho)$.

Since $\mathcal{AL}_2(\rho)$ uses only free wavelength channels by assumption, A1 is satisfied.

For any link e on $\mathcal{AL}_2(\rho)$, $\mathcal{BL}_2(\rho, e)$ is either $\mathcal{AL}_1(\rho)$ (when e is not on $\mathcal{AL}_1(\rho)$) or $\mathcal{BL}_1(\rho, e)$ (when e is on $\mathcal{AL}_1(\rho)$). Since $\mathcal{BL}_1(\rho, e)$ is the backup path for link e when e is on $\mathcal{AL}_1(\rho)$, condition A2 is satisfied.

When $\mathcal{BL}_2(\rho, e)$ is $\mathcal{AL}_1(\rho)$, it uses only free channels. When $\mathcal{BL}_2(\rho, e)$ is $\mathcal{BL}_1(\rho, e)$, it uses only free channels since $\mathcal{BL}_2(\rho)$ form a dedicated partial path protection for $\mathcal{AL}_1(\rho)$. Therefore condition A3 is satisfied. $\square$

Theorem 3.2 says that we can use any candidate active lightpath for the current connection request, without affecting the existence of dedicated partial path protection for the active path. As a result, we can always choose to use the shortest active lightpath, leading to an efficient algorithm for establishing a lightpath connection with dedicated partial path protection listed as Algorithm 1.

Theorem 3.3. *The worst-case time complexity of Algorithm 1 is $O(Wn^2 + Wnm)$. If a lightpath connection with dedicated partial path protection exists, the algorithm finds an active lightpath $\mathcal{AL}(\rho)$ and its dedicated partial path protection $\mathcal{BL}(\rho)$; otherwise, the algorithm indicates that the request should be blocked.*

Proof. It follows from Theorem 3.2 that if there exists a lightpath connection with dedicated partial path protection then any candidate active lightpath is dedicated partial path protectable. Therefore we use the shortest lightpath on free wavelength channels as the candidate active path. If such a lightpath cannot be found, a lightpath connection with dedicated partial path protection does not exist.

Once the candidate active lightpath $\mathcal{AL}(\rho)$ is found, the algorithm tries to find a low cost (measured by the number of free channels to be used) backup path for each

Algorithm 1 LPDPPP

INPUT: Network $G(V, E, \Lambda)$ with known $\mathcal{AC}(e)$ and $\mathcal{BC}(e)$ for each link $e \in E$; A connection request ρ with source $s(\rho)$ and destination $d(\rho)$.

OUTPUT: Either block the request or establish an active lightpath $\mathcal{AL}(\rho)$ and its dedicated partial path protections $\mathcal{BL}(\rho)$.

```
step_1 {Find shortest active path $\mathcal{AL}(\rho)$}
    Find a minimum hop $s(\rho)$–$d(\rho)$ lightpath $\mathcal{AL}(\rho)$ using free wavelength channels only.
    if $\mathcal{AL}(\rho)$ cannot be found then
      stop , block the request.
    else
      goto the next step, still treating the channels on $\mathcal{AL}(\rho)$ as free.
    endif
step_2 {Find dedicated PPP $\mathcal{BL}(\rho)$}
    Set $\mathcal{BL}(\rho) := \emptyset$.
    for each link $e \in \mathcal{AL}(\rho)$ do
      Set $G'$ to a copy of $G$ and make the following modifications on $G'$:
      Set the cost of each free channel not on $\mathcal{AL}(\rho)$ to 1. Set the cost of each channel on $\mathcal{AL}(\rho)$ or a backup path in $\mathcal{BL}(\rho)$ to 0. Remove all channels on link $e$ and all active channels and reserved channels.
      Find a minimum cost $s(\rho)$–$d(\rho)$ lightpath $\mathcal{BL}(\rho, e)$ in $G'$.
      if such a path does not exist then
        stop , block the request.
      elseif $\mathcal{BL}(\rho, e) \notin \mathcal{BL}(\rho)$ then
        $\mathcal{BL}(\rho) := \mathcal{BL}(\rho) \cup \{\mathcal{BL}(\rho, e)\}$.
      endif
    endfor
step_3 {Making reservations}
    for each channel $e^{\lambda}$ on $\mathcal{AL}(\rho)$ do
      mark the channel $e^{\lambda}$ as active. $\mathcal{AC}(e) := \mathcal{AC}(e) \cup \{\rho\}$.
      for each channel $f^{\sigma} \in \mathcal{BL}(\rho, e)$, $f^{\sigma} \notin \mathcal{AL}(\rho)$ do
        mark $f^{\sigma}$ as reserved. $\mathcal{BC}(f) := \mathcal{BC}(f) \cup \{\rho\}$.
      endfor
    endfor
    output$\mathcal{AL}(\rho)$ and $\mathcal{BL}(\rho)$ as the active lightpath and its dedicated partial path protections.
```

channel on $\mathcal{AL}(\rho)$. Again it follows from Theorem 3.2 that $\mathcal{BL}(\rho)$ can be computed if and only if it exists. This proves the correctness of the algorithm.

To analyze the time complexity, we note that **step_1** requires $O(Wn+Wm)$ time using bread first search [5]. **step_2** loops $O(n)$ times, each time taking $O(Wn+Wm)$ time as a shortest path can be found in linear time when the edge cost takes 0 and 1 values [5]. Therefore the time complexity of **step_2** is $O(Wn^2+Wnm)$). **step_3** only requires $O(n^2)$ time. Therefore the worst-case time complexity of Algorithm 1 is $O(Wn^2+Wnm)$. □

We wish to emphasize again that if PPP is not available for some chosen active path, then the connection request should be blocked because in that case there exists no active path with the required PPP.

We next formally define three other network models and state in each case a theorem similar to Theorem 3.2. Proofs of these theorems are similar to the proof of Theorem 3.2. For the sake of completeness, these proofs are included in the Appendix. These theorems lead to efficient algorithms for finding an active path and the corresponding PPP under the respective network models. We will describe the algorithm for LPSPPP and omit the others.

Definition 3.4. [**Lightpath Connection with Shared Partial Path Protection (LPSPPP)**] Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. A lightpath connection with shared partial path protection for ρ consists of an active path $\mathcal{AL}(\rho)$ and a set of backup paths $\mathcal{BL}(\rho)$ corresponding to $\mathcal{AL}(\rho)$, where $\mathcal{AL}(\rho)$ is a lightpath connecting $s(\rho)$ and $d(\rho)$, $\mathcal{BL}(\rho)$ is a set of lightpaths each connecting $s(\rho)$ and $d(\rho)$ such that the following conditions are satisfied:

B1: The lightpath $\mathcal{AL}(\rho)$ uses free wavelength channels only.

B2: For each link e on $\mathcal{AL}(\rho)$, there is a corresponding lightpath $\mathcal{BL}(\rho,e) \in \mathcal{BL}(\rho)$ such that $\mathcal{BL}(\rho,e)$ does not use link e. $\mathcal{BL}(\rho,e)$ is the backup path of link e on $\mathcal{AL}(\rho)$. $\mathcal{BL}(\rho,e)$ may share channels with $\mathcal{AL}(\rho)$. Also, $\mathcal{BL}(\rho,e_1)$ may share channels with $\mathcal{BL}(\rho,e_2)$ for two different links e_1 and e_2 on $\mathcal{AL}(\rho)$.

B3: Every lightpath in $\mathcal{BL}(\rho)$ uses either free wavelength channels or reserved wavelength channels.

B4: Let $\mathcal{AL}(\sigma)$ be the active path of a connection request σ that was established earlier and still in use that shares a link e with $\mathcal{AL}(\rho)$ (i.e., $\sigma \in \mathcal{AC}(e)$). Then $\mathcal{BL}(\rho,e)$ and $\mathcal{BL}(\sigma,e)$ do not share a channel.

Let $\mathcal{AL}(\rho)$ be an $s(\rho)$–$d(\rho)$ lightpath using only free wavelength channels. We say that lightpath $\mathcal{AL}(\rho)$ is shared partial path protectable if there exists a set of backup paths $\mathcal{BL}(\rho)$ such that conditions B1–B4 are satisfied. In this case, we say that $\mathcal{BL}(\rho)$ is the shared partial path protection of active lightpath $\mathcal{AL}(\rho)$.

Theorem 3.5. *Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. Let $\mathcal{AL}_1(\rho)$ and $\mathcal{AL}_2(\rho)$ be two $s(\rho)$–$d(\rho)$ lightpaths using only free wavelength channels. If lightpath $\mathcal{AL}_1(\rho)$ is shared partial path protectable then lightpath $\mathcal{AL}_2(\rho)$ is also shared partial path protectable.*

Proof. Assume that the set of lightpaths $\mathcal{BL}_1(\rho)$ is the shared partial path protection for $\mathcal{AL}_1(\rho)$. We will show that there exists a set of lightpaths $\mathcal{BL}_2(\rho)$ which is a shared partial path protection for $\mathcal{AL}_2(\rho)$.

We define $\mathcal{BL}_2(\rho)$ to be the set of lightpaths $\{\mathcal{BL}_2(\rho,e) | e \in \mathcal{AL}_2(\rho)\}$ with $\mathcal{BL}_2(\rho,e)$ defined in the following.

Let e be any link on $\mathcal{AL}_2(\rho)$. If e is not on $\mathcal{AL}_1(\rho)$, we define $\mathcal{BL}_2(\rho,e) = \mathcal{AL}_1(\rho)$. If e is on $\mathcal{AL}_1(\rho)$, we define $\mathcal{BL}_2(\rho,e) = \mathcal{BL}_1(\rho,e)$. We need to show that $\mathcal{AL}_2(\rho)$ and $\mathcal{BL}_2(\rho)$ satisfy conditions B1-B4 in Definition 3.4, i.e., $\mathcal{BL}_2(\rho)$ is a shared partial path protection for lightpath $\mathcal{AL}_2(\rho)$.

Since $\mathcal{AL}_2(\rho)$ uses only free wavelength channels by assumption of the theorem, B1 is satisfied.

For any link e on $\mathcal{AL}_2(\rho)$, $\mathcal{BL}_2(\rho, e)$ is either $\mathcal{AL}_1(\rho)$ (when e is not on $\mathcal{AL}_1(\rho)$) or $\mathcal{BL}_1(\rho, e)$ (when e is on $\mathcal{AL}_1(\rho)$). Since $\mathcal{BL}_1(\rho, e)$ is the backup path for link e when e is on $\mathcal{AL}_1(\rho)$, condition B2 is satisfied.

When $\mathcal{BL}_2(\rho, e)$ is $\mathcal{AL}_1(\rho)$, it uses only free wavelength channels. When $\mathcal{BL}_2(\rho, e)$ is $\mathcal{BL}_1(\rho, e)$, it uses either free wavelength channels or reserved wavelength channels. Therefore condition B3 is satisfied.

To complete the proof of the theorem, we need to prove that condition B4 is also satisfied. Let $\mathcal{AL}(\sigma)$ be the active path of a connection request σ that was established earlier and still in use that shares link e with $\mathcal{AL}_2(\rho)$. We need to show that $\mathcal{BL}(\sigma, e)$ and $\mathcal{BL}_2(\rho, e)$ do not share a channel. If $\mathcal{BL}_2(\rho, e) = \mathcal{AL}_1(\rho)$, then it uses only free wavelength channels and therefore does not share a channel with $\mathcal{BL}(\sigma, e)$. If $\mathcal{BL}_2(\rho, e) = \mathcal{BL}_1(\rho, e)$, then $\mathcal{BL}_1(\rho, e)$ does not share a channel with $\mathcal{BL}(\sigma, e)$ by assumption that $\mathcal{BL}_1(\rho)$ is the partial path protection of $\mathcal{AL}_1(\rho)$. Therefore $\mathcal{BL}_2(\rho, e)$ does not share a channel with $\mathcal{BL}(\sigma, e)$. $\square$

Theorem 3.5 says that we can use any candidate active lightpath for the current connection request, without affecting the existence of shared PPP for the active path. As a result, we can always choose to use the shortest active lightpath, leading to an efficient algorithm for establishing a lightpath connection with shared PPP listed as Algorithm 2.

Algorithm 2 first finds a shortest active path $\mathcal{AL}(\rho)$. When the active path cannot be found, the connection request is blocked. Once the active path is found, the algorithm then finds a low cost shared partial path protection $\mathcal{BL}(\rho)$ for $\mathcal{AL}(\rho)$. When the shared partial path protection cannot be found, the connection request is also blocked, because there does not exist an active path for the given connection request which is partial path protectable. If $\mathcal{BL}(\rho)$ is found successfully, the algorithm makes reservations and admit the connection request.

Theorem 3.6. *The worst-case time complexity of Algorithm 2 is* $O(Wn^2 + Wnm)$. *If a lightpath connection with shared partial path protection exists, the algorithm finds an active lightpath* $\mathcal{AL}(\rho)$ *and its shared partial path protection* $\mathcal{BL}(\rho)$*; otherwise, the algorithm indicates that the request should be blocked.*

Proof. It follows from Theorem 3.5 that if there exists a lightpath connection with shared partial path protection then any candidate active lightpath is shared partial path protectable. Therefore we can choose to use the shortest lightpath on free wavelength channels as the candidate active path. If such a lightpath cannot be found, a lightpath connection with shares partial path protection does not exist.

Once the candidate active lightpath $\mathcal{AL}(\rho)$ is found, the algorithm tries to find a low cost (measured by the number of free channels to be used) backup path for each channel on $\mathcal{AL}(\rho)$. Again it follows from Theorem 3.5 that $\mathcal{BL}(\rho)$ can be computed if and only if it exists. This proves the correctness of the algorithm.

To analyze the time complexity, we note that **step_1** requires $O(mW+nW)$ time using bread first search [5]. **step_2** loops $O(n)$ times, each time taking $O(mW+nW)$ time as a shortest path can be found in linear time when the edge cost takes 0 and 1 values [5]. Therefore the time complexity of **step_2** is $O(nmW + n^2W))$. **step_3** only requires $O(n^2)$ time. Therefore the worst-case time complexity of Algorithm 2 is $O(nmW + n^2W)$. $\square$

Algorithm 2 LPSPPP

```
 INPUT: Network $G(V, E, \Lambda)$ with known $\mathcal{AC}(e)$ and $\mathcal{BC}(e)$ for each link $e \in E$;
        A connection request $\rho$ with source $s(\rho)$ and destination $d(\rho)$.
OUTPUT: Either block the request or establish an active lightpath $\mathcal{AL}(\rho)$ and its
        shared partial path protection $\mathcal{BL}(\rho)$.
```

```
step_1 {Find shortest active path $\mathcal{AL}(\rho)$}
        Find a minimum hop $s(\rho)$–$d(\rho)$ lightpath $\mathcal{AL}(\rho)$ using free wavelength channels only.
        if $\mathcal{AL}(\rho)$ cannot be found then
          stop , block the request.
        else
          goto the next step, still treating the channels on $\mathcal{AL}(\rho)$ as free.
        endif
step_2 {Find shared PPP $\mathcal{BL}(\rho)$}
        Let $\mathcal{BL}(\rho) := \emptyset$.
        for each link $e \in \mathcal{AL}(\rho)$ do
          Let $G'$ be a copy of $G$ and make the following modifications on $G'$:
          Set the cost of each reserved channel to 0. Set the cost of each channel
          on $\mathcal{AL}(\rho)$ or a backup path in $\mathcal{BL}(\rho)$ to 0. Set the cost of each free
          channel not on $\mathcal{AL}(\rho)$ to 1. Remove all channels on link $e$ and all active
          channels. Remove all channels on $\mathcal{BL}(\sigma, e)$ for each $\sigma \in \mathcal{AC}(e)$.
          Find a minimum cost $s(\rho)$–$d(\rho)$ lightpath $\mathcal{BL}(\rho, e)$ in $G'$.
          if such a path does not exist then
            stop , block the request.
          elseif $\mathcal{BL}(\rho, e) \notin \mathcal{BL}(\rho)$ then
            $\mathcal{BL}(\rho) := \mathcal{BL}(\rho) \cup \{\mathcal{BL}(\rho, e)\}$.
          endif
        endfor
step_3 {Making reservations}
        for each channel $e^\lambda$ on $\mathcal{AL}(\rho)$ do
          mark the channel $e^\lambda$ as active. $\mathcal{AC}(e) := \mathcal{AC}(e) \cup \{\rho\}$.
          for each channel $f^\sigma \in \mathcal{BL}(\rho, e)$, $f^\sigma \notin \mathcal{AL}(\rho)$ do
            mark $f^\sigma$ as reserved. $\mathcal{BC}(f) := \mathcal{BC}(f) \cup \{\rho\}$.
          endfor
        endfor
        output$\mathcal{AL}(\rho)$ and $\mathcal{BL}(\rho)$ as the active lightpath and its shared partial
      path protections.
```

Definition 3.7. [**Semilightpath Connection with Dedicated Partial Path Protection (SLPDPPP)**] Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. A semilightpath connection with dedicated partial path protection for ρ consists of an active path $\mathcal{ASL}(\rho)$ and a set of backup paths $\mathcal{BSL}(\rho)$ corresponding to $\mathcal{ASL}(\rho)$, where $\mathcal{ASL}(\rho)$ is a semilightpath connecting $s(\rho)$ and $d(\rho)$, $\mathcal{BSL}(\rho)$ is a set of semilightpaths each connecting $s(\rho)$ and $d(\rho)$ such that the following conditions are satisfied:

C1: The semilightpath $\mathcal{ASL}(\rho)$ uses free wavelength channels only.

C2: For each link e on $\mathcal{ASL}(\rho)$, there is a corresponding semilightpath $\mathcal{BSL}(\rho, e) \in \mathcal{BSL}(\rho)$ such that $\mathcal{BSL}(\rho, e)$ does not use link e. $\mathcal{BSL}(\rho, e)$ is the backup path of link e on $\mathcal{ASL}(\rho)$. $\mathcal{BSL}(\rho, e)$ may share channels with $\mathcal{ASL}(\rho)$. Also, $\mathcal{BSL}(\rho, e_1)$ may share channels with $\mathcal{BSL}(\rho, e_2)$ for two different links e_1 and e_2 on $\mathcal{ASL}(\rho)$.

C3: Every semilightpath in $\mathcal{BSL}(\rho)$ uses only free wavelength channels.

Let $\mathcal{ASL}(\rho)$ be an $s(\rho)$–$d(\rho)$ semilightpath using only free wavelength channels. We say that semilightpath $\mathcal{ASL}(\rho)$ is dedicated partial path protectable if there exists a set of backup paths $\mathcal{BSL}(\rho)$ such that conditions C1–C3 are satisfied. In this case, we say that $\mathcal{BSL}(\rho)$ is the dedicated partial path protection of active semilightpath $\mathcal{ASL}(\rho)$.

Theorem 3.8. *Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. Let $\mathcal{ASL}_1(\rho)$ and $\mathcal{ASL}_2(\rho)$ be two s–d semilightpaths using only free wavelength channels. If there exists a set of semilightpaths $\mathcal{BSL}_1(\rho)$ so that $\mathcal{ASL}_1(\rho)$ and $\mathcal{BSL}_1(\rho)$ form a semilightpath connection with dedicated partial path connection for ρ with $\mathcal{ASL}_1(\rho)$ as the active path, then there exists a set of semilightpaths $\mathcal{BSL}_2(\rho)$ so that $\mathcal{ASL}_2(\rho)$ and $\mathcal{BSL}_2(\rho)$ form a semilightpath connection with dedicated partial path connection for ρ with $\mathcal{ASL}_2(\rho)$ as the active path. In other words, $\mathcal{ASL}_1(\rho)$ is dedicated partial path protectable if and only if $\mathcal{ASL}_2(\rho)$ is dedicated partial path protectable.*

Proof. We define $\mathcal{BSL}_2(\rho)$ to be the set $\{\mathcal{BSL}_2(\rho, e) | e \in \mathcal{ASL}_2(\rho)\}$ with $\mathcal{BSL}_2(\rho, e)$ defined in the following.

Let e be any link on $\mathcal{ASL}_2(\rho)$. If e is not on $\mathcal{ASL}_1(\rho)$, we define $\mathcal{BSL}_2(\rho, e) = \mathcal{ASL}_1(\rho)$. If e is on $\mathcal{ASL}_1(\rho)$, we define $\mathcal{BSL}_2(\rho, e) = \mathcal{BSL}_1(\rho, e)$. We need to show that $\mathcal{ASL}_2(\rho)$ and $\mathcal{BSL}_2(\rho)$ satisfy conditions C1-C3 in Definition 3.7, i.e., $\mathcal{BSL}_2(\rho)$ is a dedicated partial path protection of $\mathcal{ASL}_2(\rho)$.

Since $\mathcal{ASL}_2(\rho)$ uses only free wavelength channels by assumption, C1 is satisfied.

For any link e on $\mathcal{ASL}_2(\rho)$, $\mathcal{BSL}_2(\rho, e)$ is either $\mathcal{ASL}_1(\rho)$ (when e is not on $\mathcal{ASL}_1(\rho)$) or $\mathcal{BSL}_1(\rho, e)$ (when e is on $\mathcal{ASL}_1(\rho)$). Since $\mathcal{BSL}_1(\rho, e)$ is the backup path for link e when e is on $\mathcal{ASL}_1(\rho)$, condition C2 is satisfied.

When $\mathcal{BSL}_2(\rho, e)$ is $\mathcal{ASL}_1(\rho)$, it uses only free channels. When $\mathcal{BSL}_2(\rho, e)$ is $\mathcal{BSL}_1(\rho, e)$, it uses only free channels since $\mathcal{BSL}_2(\rho)$ form a dedicated partial path protection for $\mathcal{ASL}_1(\rho)$. Therefore condition C3 is satisfied. ☐

Theorem 3.8 immediately leads to a polynomial time algorithm for establishing a semilightpath connection with dedicated partial path protection, which uses the shortest candidate active semilightpath. The algorithm is similar to Algorithm 1 and is omitted here.

Definition 3.9. [**Semilightpath Connection with Shared Partial Path Protection (SLPSPPP)**] Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. A semilightpath connection with shared partial path protection for ρ

consists of an active path $\mathcal{ASL}(\rho)$ and a set of backup paths $\mathcal{BSL}(\rho)$ corresponding to $\mathcal{ASL}(\rho)$, where $\mathcal{ASL}(\rho)$ is a semilightpath connecting $s(\rho)$ and $d(\rho)$ and $\mathcal{BSL}(\rho)$ is a set of semilightpaths each connecting $s(\rho)$ and $d(\rho)$ such that the following conditions are satisfied:

D1: The semilightpath $\mathcal{ASL}(\rho)$ uses free wavelength channels only.

D2: For each link e on $\mathcal{ASL}(\rho)$, there is a corresponding semilightpath $\mathcal{BSL}(\rho, e) \in \mathcal{BSL}(\rho)$ such that $\mathcal{BSL}(\rho, e)$ does not use link e. $\mathcal{BSL}(\rho, e)$ is the backup path of link e on $\mathcal{ASL}(\rho)$. $\mathcal{BSL}(\rho, e)$ may share channels with $\mathcal{ASL}(\rho)$. Also, $\mathcal{BSL}(\rho, e_1)$ may share channels with $\mathcal{BSL}(\rho, e_2)$ for two different links e_1 and e_2 on $\mathcal{ASL}(\rho)$.

D3: Every semilightpath in $\mathcal{BSL}(\rho)$ uses either free wavelength channels or reserved wavelength channels.

D4: Let $\mathcal{ASL}(\sigma)$ be the active path of a connection request σ that was established earlier and still in use that shares a link e with $\mathcal{ASL}(\rho)$ (i.e., $\sigma \in \mathcal{AC}(e)$). Then $\mathcal{BSL}(\rho, e)$ and $\mathcal{BSL}(\sigma, e)$ do not share a channel.

Let $\mathcal{ASL}(\rho)$ be an $s(\rho)$–$d(\rho)$ semilightpath using only free wavelength channels. We say that semilightpath $\mathcal{ASL}(\rho)$ is shared partial path protectable if there exists a set of backup paths $\mathcal{BSL}(\rho)$ such that conditions D1–D4 are satisfied. In this case, we say that $\mathcal{BSL}(\rho)$ is the shared partial path protection of active semilightpath $\mathcal{ASL}(\rho)$.

Theorem 3.10. *Let ρ be a connection request with source $s(\rho)$ and destination $d(\rho)$. Let $\mathcal{ASL}_1(\rho)$ and $\mathcal{ASL}_2(\rho)$ be two s–d semilightpaths using only free wavelength channels. If there exists a set of semilightpaths $\mathcal{BSL}_1(\rho)$ so that $\mathcal{ASL}_1(\rho)$ and $\mathcal{BSL}_1(\rho)$ form a semilightpath connection with shared partial path connection for ρ with $\mathcal{ASL}_1(\rho)$ as the active path, then there exists a set of semilightpaths $\mathcal{BSL}_2(\rho)$ so that $\mathcal{ASL}_2(\rho)$ and $\mathcal{BSL}_2(\rho)$ form a semilightpath connection with shared partial path connection for ρ with $\mathcal{ASL}_2(\rho)$ as the active path.*

Proof. We define $\mathcal{BSL}_2(\rho)$ to be the set $\{\mathcal{BSL}_2(\rho, e) | e \in \mathcal{ASL}_2(\rho)\}$ with $\mathcal{BSL}_2(\rho, e)$ defined in the following.

Let e be any link on $\mathcal{ASL}_2(\rho)$. If e is not on $\mathcal{ASL}_1(\rho)$, we define $\mathcal{BSL}_2(\rho, e) = \mathcal{ASL}_1(\rho)$. If e is on $\mathcal{ASL}_1(\rho)$, we define $\mathcal{BSL}_2(\rho, e) = \mathcal{BSL}_1(\rho, e)$. We need to show that $\mathcal{ASL}_2(\rho)$ and $\mathcal{BSL}_2(\rho)$ satisfy conditions D1-D4 in Definition 3.9, i.e., $\mathcal{BSL}_2(\rho)$ is a shared partial path protection for semilightpath $\mathcal{ASL}_2(\rho)$.

Since $\mathcal{ASL}_2(\rho)$ uses only free wavelength channels by assumption, D1 is satisfied.

For any link e on $\mathcal{ASL}_2(\rho)$, $\mathcal{BSL}_2(\rho, e)$ is either $\mathcal{ASL}_1(\rho)$ (when e is not on $\mathcal{ASL}_1(\rho)$) or $\mathcal{BSL}_1(\rho, e)$ (when e is on $\mathcal{ASL}_1(\rho)$). Since $\mathcal{BSL}_1(\rho, e)$ is the backup path for link e when e is on $\mathcal{ASL}_1(\rho)$, condition D2 is satisfied.

When $\mathcal{BSL}_2(\rho, e)$ is $\mathcal{ASL}_1(\rho)$, it uses only free channels. When $\mathcal{BSL}_2(\rho, e)$ is $\mathcal{BSL}_1(\rho, e)$, it uses either free channels or reserved channels. Therefore condition D3 is satisfied.

To complete the proof of the theorem, we need to prove that condition D4 is also satisfied. Let $\mathcal{ASL}(\sigma)$ be the active path of a connection request σ that was established earlier and still in use that shares link e with $\mathcal{ASL}_2(\rho)$. We need to show that $\mathcal{BSL}(\sigma, e)$ and $\mathcal{BSL}_2(\rho, e)$ do not share a channel. If $\mathcal{BSL}_2(\rho, e) = \mathcal{ASL}_1(\rho)$, then it uses only free wavelength channels and therefore does not share a channel with $\mathcal{BSL}(\sigma, e)$. If $\mathcal{BSL}_2(\rho, e) = \mathcal{BSL}_1(\rho, e)$, then $\mathcal{BSL}_1(\rho, e)$ does not share a channel with $\mathcal{BSL}(\sigma, e)$ by assumption that $\mathcal{BSL}_1(\rho, e)$ is the partial path protection of $\mathcal{ASL}_1(\rho)$. Therefore $\mathcal{BSL}_2(\rho, e)$ does not share a channel with $\mathcal{BSL}(\sigma, e)$. □

Theorem 3.10 immediately leads to a polynomial time algorithm for establishing a semilightpath connection with shared partial path protection. The algorithm is similar to Algorithm 2 and is omitted here.

4. Relationships with Segmented Path Protection.

Almost at the same time when partial path protection was proposed, several researchers also proposed the *segmented path protection scheme* [12, 25], the *short leap shared protection* [13], and *PROMISE* [32, 33] as extensions of the traditional path protection scheme. These schemes have been shown to have good performance in terms of spare capacity usage. We would like to point out that no polynomial time algorithm (except some heuristics) is known for any of these schemes, while PPP enjoys polynomial time algorithms. We will not go into further details of all these schemes. In this section, we will briefly compare segmented path protection with PPP.

The idea of segmented path protection is to break the active path into primary segments and provide protection segment for each of the segments. *When a component in a primary segment fails, the data is routed through the protection segment activated rather than through the original path, only for the length of its primary segment* [25]. On first reading, it seems that partial path protection is equivalent to segmented path protection. In this section, we will show that they are indeed different and that partial path protection has advantages over segmented path protection.

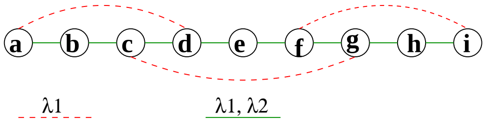


FIGURE 3. a-b-c-d-e-f-g-h-i on λ_1 is an active lightpath which does not have a link-disjoint backup lightpath, but can be break into primary segments a-b-c-d, c-d-e-f-g, and f-g-h-i, with corresponding protection segments a-d on λ_1, c-g on λ_1, and f-i on λ_1, respectively.

We will use Fig. 3 to illustrate the concept of segmented path protection. We note that a-b-c-d-e-f-g-h-i on λ_1 is an active lightpath which does not have a link-disjoint backup lightpath. However, it can be break into primary segments a-b-c-d, c-d-e-f-g, and f-g-h-i, with corresponding protection segments a-d on λ_1, c-g on λ_1, and f-i on λ_1, respectively. If any one of the links in $\{(a,b),(b,c),(c,d)\}$ fails, the protection segment a-d on λ_1 is activated and traffic on the active path may be shifted onto the lightpath a-d-e-f-g-h-i on λ_1. If any one of the links in $\{(c,d),(d,e),(e,f),(f,g)\}$ fails, the protection segment c-g on λ_1 is activated and traffic on the active path may be shifted onto the lightpath a-b-c-g-h-i on λ_1. If any one of the links in $\{(f,g),(g,h),(h,i)\}$ fails, the protection segment f-i on λ_1 is activated and traffic on the active path may be shifted onto the lightpath a-b-c-d-e-f-i on λ_1.

We note that the active lightpath a-b-c-d-e-f-g-h-i on λ_1 is also partial path protectable and its partial path protection can be easily constructed from its segmented path protection. For example, for each of the links a-b, b-c and c-d on the active lightpath, we can use the lightpath a-d-e-f-g-h-i on λ_1 as its partial path protection; for each of the links d-e and e-f on the active lightpath, we can use the lightpath *a*-*b*-*c*-*g*-*h*-*i* on λ_1 as its partial path protection; for each of the links *f*-*g*, *g*-*h* and *h*-*i* on the active lightpath, we can use the lightpath *a*-*b*-*c*-*d*-*e*-*f*-*i* on λ_1 as its partial path protection.

In fact we can prove that the existence of segmented path protection for a given active lightpath implies that the corresponding active lightpath is partial path protectable. The reverse is not true, as shown in the following example.

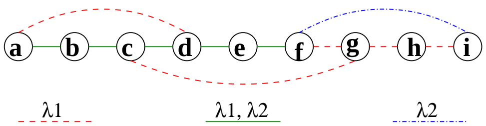


FIGURE 4. *a*-*b*-*c*-*d*-*e*-*f*-*g*-*h*-*i* on λ_1 is an active lightpath which is partial path protectable, but not segmented path protectable.

The active lightpath *a*-*b*-*c*-*d*-*e*-*f*-*g*-*h*-*i* on wavelength λ_1 in Fig. 4 is partial path protectable, with links *a*-*b*, *b*-*c*, *c*-*d* protected by lightpath *a*-*d*-*e*-*f*-*g*-*h*-*i* on wavelength λ_1; links *d*-*e*, *e*-*f* protected by lightpath *a*-*b*-*c*-*g*-*h*-*i* on wavelength λ_1; and links *f*-*g*, *g*-*h*, *h*-*i* protected by lightpath *a*-*b*-*c*-*d*-*e*-*f*-*i* on wavelength λ_2. However, the active lightpath *a*-*b*-*c*-*d*-*e*-*f*-*g*-*h*-*i* on wavelength λ_1 does not have segmented path protection. This is so because there are only three possible protection segments: *a*-*d* on λ_1, *c*-*g* on λ_1, and *f*-*i* on λ_2. We note that the protection segment *f*-*i* only covers the primary segment *f*-*g*-*h*-*i*. If link *h*-*i* fails, the segmented path protection scheme [25] would only active the segment *f*-*i* on wavelength λ_2. Note that the part of the active path *a*-*b*-*c*-*d*-*e*-*f* (on wavelength λ_1) and the protection segment *f*-*i* (on wavelength λ_2) are on two wavelengths. Therefore their concatenation is not a lightpath! This shows that the active lightpath *a*-*b*-*c*-*d*-*e*-*f*-*g*-*h*-*i* on wavelength λ_1 does not have a segmented path protection in a network without wavelength converters.

Detailed comparisons of the path protection scheme, the segmented path protection scheme and the partial path protection scheme will be presented in a future paper.

5. **Conclusions and Future Work.** In this paper, we have studied survivable routing in WDM networks using the partial path protection schemes. Depending on whether protection is shared or dedicated, and wavelength continuity constraint is required or not, we have formulated and studied four different problems. These are lightpath connection with dedicated partial path protection (LPDPPP), lightpath connection with shared partial path protection (LPSPPP), semilightpath connection with dedicated partial path protection (SLPDPPP), and semilightpath connection with shared partial path protection (SLPSPPP). For each of the four problems, we have proved that if a candidate active path has partial path protection then every other candidate active path also has partial path protection. Therefore it is reasonable to use the shortest path on free channels as the candidate active path. As a result, an active path and its corresponding partial path protection can be computed in polynomial time as long as they exist.

We also wish to emphasize the significance of the work in this paper in the context of polynomial time computability of path selection problems arising in different path/segmented path-based schemes. First, there is no known polynomial time algorithm for computing an active path and a corresponding backup path using the path protection scheme for a given source-destination pair, except for the case of dedicated path protection in a WDM network with wavelength converters, in which case the problem becomes one of computing a pair of link disjoint paths in a graph [27]. In fact, it has been shown in [2, 20] that computing a lightpath and a corresponding backup path when wavelength converters are not allowed is NP-hard in both the dedicated path protection case and the shared path protection case. Also, there is no known polynomial time algorithm for computing an active path and a corresponding segmented path protection, except in the case of dedicated segmented path protection in a network with wavelength converters at all nodes where again the problem becomes one of computing a pair of link disjoint paths in a graph. On the other hand, we have shown in Section 3 that the existence or otherwise of an active path and a corresponding set of backup paths using the partial protection scheme can be determined in polynomial time. We have also shown that if a set of backup paths (for partial path protection) exists for an active path, then a set of backup paths exists for any active path. This allows us the freedom to select paths (active as well as backup) which satisfy certain QoS guarantees and opens up new avenues for further enhancement of protection schemes for WDM networks. This is particularly significant since there are a large number of heuristic/approximation algorithms reported in the literature for the QoS routing problem.

It has been shown in [30] that the existence of PP implies the existence of PPP while the reverse is not true. We have noted in Section 4 that the existence of segmented path protection for a given active lightpath implies that the corresponding active lightpath is partial path protectable, while the reverse is not true. We conclude noting that further research is called for on the relationships among the different path (including segmented path) based protection schemes with regards to their relative power in providing protection. We have already made considerable progress in this direction and will report our results in a companion paper.

REFERENCES

[1] V. Anand and C. Qiao, *Dynamic establishment of protection paths in WDM networks*, Part I, IEEE ICCCN'2000, 198–204.

[2] R. Andersen, F. Chung, A. Sen and G. Xue, *On disjoint path pairs with wavelength continuity constraint in WDM networks*, IEEE INFOCOM'2004, 524–535.

[3] I. Chlamtac, A. Faragó and T. Zhang, *Lightpath (wavelength) routing in large WDM networks*, IEEE JSAC, **14** (1996), 909–913.

[4] I. Chlamtac, A. Ganz and G. Karmi, *Lightpath communications: an approach to high bandwidth optical WAN's*, IEEE Trans. on Communications, **40** (1992), 1171–1182.

[5] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein, "Introduction to Algorithms," 2nd edition, MIT Press, Cambridge, Massachusetts, 2001.

[6] G. Ellinas, G. Halemariam and T. Stern, *Protection cycles in mesh WDM networks*, IEEE JSAC, **18** (2000), 1924–1937.

[7] M. T. Frederick and A. K. Somani, *A single-fault recovery strategy for optical networks using subgraph routing*, IFIP ONDM, (2003), 327–346.

[8] A. Fumagalli and L. Valcarenghi, *IP restoration versus WDM protection: is there an optimal choice?* IEEE Networks, **14** (2000), 34–41.

[9] P. E. Green and Jr., "Fiber Optic Networks," Prentice-Hall, Inc., 1993.

[10] W. D. Grover and D. Stamatelakis, *Cycle-oriented distributed preconfiguration: ring-like speed with mesh-like capacity for self-planning network configuration*, IEEE ICC'1998, 537–543.

[11] W. D. Grover and D. Stamatelakis, *IP layer restoration and network planning based on virtual protection cycles*, IEEE JSAC, **18** (2000), 1938–1949.

[12] K. P. Gummadi, M. J. Pradeep and C. S. R. Murthy, *An efficient primary-segmented backup scheme for dependable real-time communication networks*, IEEE/ACM Transactions on Networking, **11** (2003), 81-94.

[13] P. H. Ho and H. T. Mouftah, *A framework of service guaranteed shared protection for optical networks*, IEEE Communications Magazine, **40** (2002), 97–103.

[14] A. Itai and M. Rodeh, *The multi-tree approach to reliability in distributed networks*, Information and Computation, **79** (1988), 43–59.

[15] Y. Liu and D. Tipper, *Spare capacity allocation for non-linear cost and failure dependent path restoration*, DRCN'2001, Budapest, Hungary.

[16] M. Médard, R. A. Barry, S. G. Finn, W. He and S. S. Lumetta, *Generalized loop-back recovery in optical mesh networks*, IEEE/ACM Transactions Networking, **10** (2002), 153–164.

[17] M. Médard, S. G. Finn, R. A. Barry and R. G. Gallager, *Redundant trees for preplanned recovery in arbitrary vertex-redundant or edge-redundant graphs*, IEEE/ACM Transactions on Networking, **7** (1999), 641–652.

[18] G. Mohan, C. S. Ram Murthy and Arun Somani, *Efficient algorithms for routing dependable connections in WDM optical networks*, IEEE/ACM Transactions on Networking, **9** (2001), 553–566.

[19] B. Mukherjee, "Optical Communication Networks," McGraw Hill, 1997.

[20] C. Ou, J. Zhang, H. Zang, L. Sahaasrabuddhe and B. Mukherjee, *New and improved approaches for shared-path protection in WDM mesh networks*, IEEE/OSA Journal of Lightwave Technology, **22** (2004), 1223–1232.

[21] S. Ramamurthy and B. Mukherjee, *Survivable WDM mesh networks, Part I-Protection*, IEEE INFOCOM'1999, 744–751.

[22] S. Ramasubramanian, *On failure dependent protection in optical grooming networks*, IEEE DSN'2004, 475–484.

[23] G. Sahin and M. Azizoglu, *Optical layer survivability: single service-class case*, SPIE OPTICOMM'2000, 267–278.

[24] G. Sahin and M. Azizoglu, *Optical layer survivability for single and multiple service classes*, Journal of High Speed Networks, **10** (2001), 91–108.

[25] C. V. Saradhi and C. S. Ram Murthy, *Segmented protection paths in WDM mesh networks*, HPSR'2003, 311–316.

[26] R. Srinivasan, *MICRON: A framework for connection establishment in optical networks*, SPIE OPTICOMM'2003, 139–150.

[27] J. W. Suurballe, *Disjoint paths in a network*, Networks, **4** (1974), 125–145.

[28] B. Van Caenegem, W. Van Parys, F. De Turck and P.M. Demeester, *Dimensioning of survivable WDM networks*, IEEE JSAC, **16** (1998), 1146–1157.

[29] B. Van Caenegem, N. Wauters and P. Demeester, *Spare capacity assignment for different restoration strategies in mesh survivable networks*, IEEE ICC'1997, 288–292.

[30] H. Wang, E. Modiano and M. Médard, *Partial path protection for WDM networks: End-to-end recovery using local failure information*, IEEE ISCC'02, 719–725.

[31] Y. Xiong and L. G. Mason, *Restoration strategies and spare capacity requirements in self-healing ATM networks*, IEEE/ACM Transactions on Networking, **7** (1999), 98–110.

[32] D. Xu, Y. Xiong and C. Qiao, *Protection with multi-segments in networks with shared risk link groups (SRLG)*, 40th Annual Allerton Conference on Communications, Control, and Computing, Allerton, IL, 2002.

[33] D. Xu, Y. Xiong and C. Qiao, *Novel algorithms for shared segment protection*, IEEE JSAC-OCN, **21** (2003), 1320–1331.

[34] G. Xue, L. Chen and K. Thulasiraman, *QoS issues in redundant trees for protection in vertex-redundant or edge- redundant networks*, IEEE ICC'2002, 2766–2770.

[35] G. Xue, L. Chen and K. Thulasiraman, *Quality of service and quality of protection issues in preplanned recovery schemes using redundant trees*, IEEE JSAC-OCN, **21** (2003), 1332–1345.

[36] G. Xue and R. Gottapu, *Efficient construction of virtual p-cycles protecting all cycle-protectable working links*, IEEE HPSR'2003, 305–310.

Received October 2006; revised December 2006, April 2007 and July 2007.

E-mail address: `xue@asu.edu`
E-mail address: `weiyi.zhang@asu.edu`
E-mail address: `tie.wang@asu.edu`
E-mail address: `thulasi@ou.edu`