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Reprinted from the *Journal of the Institution of Engineers (India)*,
Vol. XLVII, No. 10, Pt. EL 5, June 1967

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Prof. S. Srinivasan
Member
Professor of Electrical Engineering, College of Engineering, Guindy, Madras.

V. N. Sujeer
Associate Member
Assistant Professor of Electrical Engineering, College of Engineering, Guindy, Madras

and

K. Thulasiraman
Non-member
Department of Electrical Engineering, Indian Institute of Technology, Madras

U.D.C. 621.311.2

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S. Srinivasan
Member
Professor of Electrical Engineering, College of Engineering, Guindy, Madras
V. N. Sujeer
Associate Member
Professor of Electrical Engineering, College of Engineering, Guindy, Madras
and
K. Thulasiraman
Non-member
Department of Electrical Engineering, Indian Institute of Technology, Madras

Summary

This paper deals with the calculation procedure called 'restoration process' for the solution of actual bus voltages and currents in a power system, utilizing the information stored during the reduction of the power system, by the new equivalence technique in linear graph theory. The process is illustrated with an example.

Notations

A = incidence matrix,
A_{mn} = partitioned submatrix of the A-matrix,
A_{mn}' = transposition of the partitioned submatrix A_{mn},
Y_D = D-graph admittance matrix,
Y_E = E-graph admittance matrix,
Y_{EG} = E-graph admittance matrix for the final generator daisy,
Y_S = admittance matrix of S-graph elements,
I_E = current matrix of E-graph elements,
V_S = node voltages of S-graph,
V_E = node voltages of E-graph,
I_S = current matrix of S-graph,
v_D = number of vertices in D-graph,
v_J = number of J vertices,
u = unit matrix,
U = U-vertex.

* **Written discussion on this paper will be received until October 31, 1967.**
This paper was received on September 21, 1966.

1. Introduction

In the reduction of a network or graph by the equivalence technique described in an earlier paper[1] the D-graph is replaced by an E-graph and in the process the individual vertices of the D-graph and associated elements are eliminated. The information details of these individual vertices and elements are, therefore, also suppressed or buried. The results of reduction give equivalent admittances or impedances across the junction vertices.

In another paper[2], the application of the reduction technique to a power system was described. The method enables one to determine the system admittances as viewed from generator terminals or important substations.

For a comprehensive or more detailed study, however, currents in the individual elements or branches and voltages at all nodes or buses are required. This requires the transference of information from the E-graph back to the D-graph and is thus a reverse of the process of reduction. Certain results and equations of the reduction process can be used to advantage for this process. Hence, if these results of the reduction process are stored, then they can be utilized for uncovering the original elements and vertices and thereby to calculate their currents and voltages by a routine procedure. This orderly process of calculation from the reduced equivalent graph, the I and V functions of the original graph is termed as 'restoration'.

This paper explains the details of the restoration process in the I explicit form and its applications to a power system. The notation used in this paper is the same as that used in the previous papers.[1,2] The method is illustrated by applying it to the power system reduced in a previous paper.[2]

2. Restoration

The SE-graph specifies the V functions of the E-graph. Since the V functions V_E of the E-graph, equal the V functions, V_N of the N-graph, the latter are also specified. To obtain all the V functions of the N_D-graph, a set of V functions corresponding to a tree of N_D is required.

Case 1 : $V_D > J_k$; S and D coupled

In this case, the N-graph does not contain V_D-1 elements and hence does not constitute a tree of N_D. To obtain the required information, some more V functions in addition to those of V_N should be known. These V functions are designated as V_{DB}. They correspond to a set of elements of the D-graph which together with the N-graph constitutes a tree of ND-graph.

The f-seg matrix of ND-graph in the tree is given by

$$\begin{array}{r} j_k - P_{sk} \\ V_D\text{-}1\text{-}j_k + P_{sk} \end{array} \begin{bmatrix} U_N & S_{12} \\ O & S_{22} \end{bmatrix} \qquad (1)$$

where U_N is the unit matrix corresponding to the N-graph.

The F-element equations for the D-graph and the elements of S which are coupled to D are expressed as

$$\begin{bmatrix} I_{SD} \\ \\ I_D \end{bmatrix} = \begin{bmatrix} Y_S & Y_{SD} \\ \\ Y_{DS} & Y_D \end{bmatrix} \begin{bmatrix} V_{SD} \\ \\ V_D \end{bmatrix} \tag{2}$$

where I_{SD} and V_{SD} contain respectively the I and V functions, which are associated with elements of S that are coupled to D, I_D and V_D contain respectively the I and V functions, which are associated with all the elements of D.

Then, from the previous paper,[3]

$$V_{DB} = [J_{DSV} \; J_{EV}] \begin{bmatrix} V_{SD} \\ V_E \end{bmatrix} \tag{3}$$

where,

$$J_{DSV} = -[S_{22} \, Y_D \, S_{22}']^{-1} \, S_{22} \, Y_{DS} \tag{4}$$

$$J_{EV} = -[S_{22} \, Y_D \, S_{22}']^{-1} \, S_{22} \, Y_D \, S_{12}' \tag{5}$$

From V_{DB} and V_E, V_D can be calculated from

$$V_D = [S_{12}' \; S_{22}'] \begin{bmatrix} V_E \\ V_{DB} \end{bmatrix} \tag{6}$$

J_{DSV}, J_{EV}, S_{12}' and S_{22}' are all determined during the reduction process. Hence, these calculated results are to be stored and made available for restoration. The calculations involve only matrix multiplication.

Case 2 : $V_D > j_k$; S and D not coupled

In this case,

$$Y_{DS} = 0$$

Hence,

$$J_{DSV} = 0$$

From equations (3) and (6),

$$V_{DB} = J_{EV} \, V_E \tag{7}$$

and

$$V_D = [S_{12}' \; S_{22}'] \begin{bmatrix} V_E \\ V_{DB} \end{bmatrix} \tag{8}$$

Cace 3 : $V_D = j_k$; S and D coupled

If $P_{sk} > 1$, the V functions of the N-graph do not constitute a tree set. In such a case,

$$V_{DB} = [J_{DSV} \; J_{EV}] \begin{bmatrix} V_{SD} \\ V_E \end{bmatrix} \tag{9}$$

and

$$V_D = [S_{12}' \text{-} S_{22}'] \begin{bmatrix} V_E \\ V_{DB} \end{bmatrix} \tag{10}$$

If the S-graph is in only one part, that is, $P_{sk} = 1$, N-graph constitutes a tree of ND-graph. Hence, $S_{22} = 0$

From equation (6),

$$V_D = S_{12}' \, V_E \tag{11}$$

Case 4 : $V_D = j_k$ *; S and D not coupled*

If $p_{sk} > 1$ then V_{DB} and V_D can be calculated from equations (7) and (8). If $p_{sk} = 1$ V_D is given by equation (11).

3. Application of restoration process to power system

The reduction of a power system is carried out by splitting the system into a convenient number of zones.[2] After eliminating the non-U-vertices, each zone is reduced to its G-U-form. The G-U-graphs of individual zones are then put together and the elimination of all the U-vertices is done to obtain the generator daisy of the system. Incidence matrix is the basis seg matrix used. The V functions are the node voltages, with respect to ground. N or E-graph is a Lagrangian tree with ground as the star point. All the elements of N and E are oriented towards ground. The incidence matrix A of ND-graph with the row corresponding to ground eliminated, has the form shown in equation (12).

$$A = \begin{array}{l} \\ J \text{ vertices} \\ \text{Non-}J \text{ vertices} \end{array} \begin{array}{c} \begin{array}{cc} N & D \end{array} \\ \begin{bmatrix} U & A_{12} \\ O & A_{22} \end{bmatrix} \end{array} \tag{12}$$

The S and D-graphs are chosen such that there is no coupling between them.

The restoration process consists in the computation of the node-voltages of each D-graph chosen in the reduction process from the calculated node voltages of its displaced E-graph. The node voltages of the E-graph are the same as the voltages of the J vertices. Hence, the only node voltages of D-graph to be restored are the voltages V_{ND} of the non-J vertices of the D-graph which are submerged during reduction. It must be noted that V_{ND} corresponds to V_{DB}. The general formula for the computation of V_{ND} is given by equation (7), with S replaced by A, *i.e.*,

$$V_{ND} = J_{EV} \, V_E = -[A_{22} \, Y_D \, A_{22}']^{-1} \, A_{22} \, Y_D \, A_{12}' \, V_E \tag{13}$$

where Y_D is the admittance matrix of the D-graph.

The all zone generator daisy of Fig. 1 is the SE-graph obtained from the all-zone G-U-graph of Fig. 2 after eliminating all the U vertices. Generator buses are the J vertices and hence generator bus voltages are the node voltages of the E-graph. The

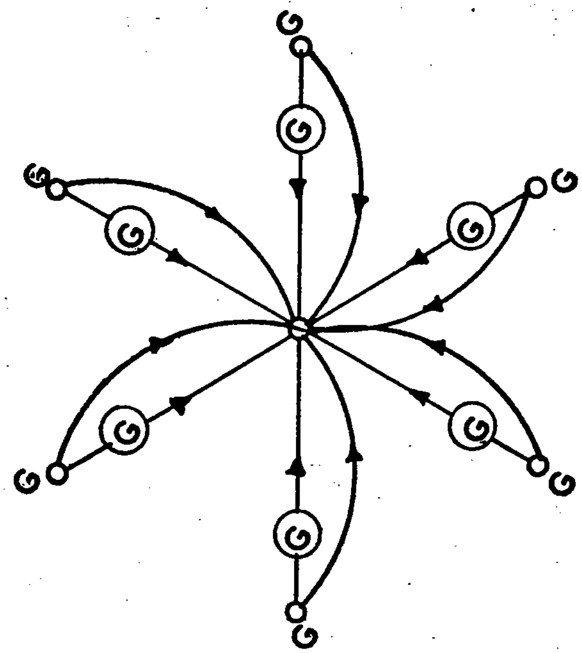


Fig. 1

All-zone generator daisy equivalent

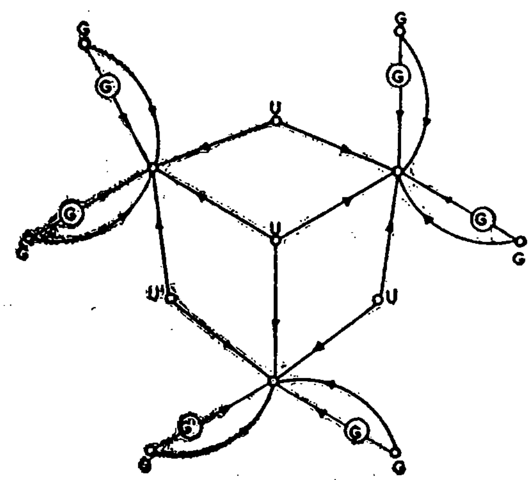


Fig. 2

All-zone G-U-Graph

first step in restoration is the computation of all the U vertex voltages. The admittance matrix of all the elements except the generaors of all-zone G-U graph is given by

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{cc} \text{Non-U-elements} & \text{U-elements} \end{array} \\ \left[\begin{array}{ccc:ccc} Y_{11,1} & & 0 & Y_{12,1} & & 0 \\ & Y_{11,2} & & & Y_{12,2} & \\ O & & Y_{11,n} & O & & Y_{12,n} \\ \hdashline Y_{21,1} & & 0 & Y_{22,1} & & 0 \\ & Y_{21,2} & & & Y_{22,2} & \\ 0 & & Y_{21,n} & 0 & & Y_{22,n} \end{array}\right] \end{array} \tag{14}$$

where $Y_{11,i}$, $Y_{12,i}$, $Y_{22,i}$ and $Y_{21,i}$ are the submatrices of $Y_{D,i}$, which is the admittance matrix of all non-generator elements of G-U graph of zone i, *i.e.*,

$$Y_{D,i} = \begin{bmatrix} Y_{11,i} & Y_{12,i} \\ Y_{21,i} & Y_{22,i} \end{bmatrix} \tag{15}$$

The column of the ND-graph incidence matrix pertaining to the D-graph, with partioning corresponding to that for equation (14) is

$$A = \begin{bmatrix} A_{12} \\ \\ A_{22} \end{bmatrix} = \begin{array}{c} \\ \text{Generator vertices} \\ \\ \text{U-vertices} \end{array} \begin{array}{c} \begin{array}{cc} \text{Elements incidents to generator vertices} & \text{U-elements} \end{array} \\ \left[\begin{array}{ccc:c} U_1 & & & 0 \\ & U_2 & & \\ & & U_n & \\ \hdashline & 0 & & A_{23} \end{array}\right] \end{array} \tag{16}$$

where if the last set of columns is ordered properly, with respect to zones then,

$$A_{23} = [A_{23,1}, \ A_{23,2} \ \ldots \ A_{23,n}] \tag{17}$$

Substituting the various submatrices of equations (14), (15), (16) and 17 in equation (13), the zonewise ordered set of node-voltage V_{ND}' of all the U-vertices can be obtained by

$$V_{ND} = -\left[[0 \quad A_{23}] \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} 0 \\ A_{23}' \end{bmatrix} \right]^{-1}$$

$$[0 \quad A_{23}] \quad \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} U \\ 0 \end{bmatrix} \quad V_E$$

$$= -[A_{23} \quad Y_{22} \quad A_{23}]^{-1} \qquad A_{23} \quad Y_{21} \quad V_E$$

$$= -[B]^{-1} [M_1', M_2' \quad \ldots \quad M_n'] \begin{bmatrix} V_{G,1} \\ V_{G,2} \\ \cdot \\ \cdot \\ \cdot \\ V_{G,n} \end{bmatrix} \tag{18}$$

where

$$B = \sum_{i=1}^{n} A_{23,i} \quad Y_{22,i} \quad A'_{23,i}$$

and

$$M_i = Y_{12,i} A'_{23,i}$$

$V_{G,i}$ is the set of generator bus voltages in zone i. B^{-1} and M_i are all already calculated during the reduction process.

The U element voltages are given by

$$V_{u,e} = A_{23}' V_{ND} \tag{19}$$

With this computation, the restoration of the all zone G-U graph is completed. The node voltage matrix of each zone G-U graph is of the form

$$V_{n,i} = \begin{bmatrix} V_{G,i} \\ V_{u,i} \end{bmatrix} \tag{20}$$

where $V_{u,i}$ is the set voltages of U-vertices in zone i. V_{ni}, is also the $V_{E,i}$ for the next step in restoration process now at the G-U level.

Using equation (20) in equation (13), *i.e.*,

$$V_{ND,i} = J_{EV,i} \, V_{E,i} \tag{21}$$

the voltages of the nodes submerged during the previous reduction of zone i can be calculated. $J_{EV,i}$ is already calculated and stored during the reduction. Continuing this process of restoration, the voltages of all the nodes of each zone can be obtained. This process restores the G-U graph of each zone (Fig. 3) to its all-vertex form (Fig. 4).

The element voltages of each zone are calculated by the equations :

$$V_{L,i} = A'_{1,i} \, V_{u,i} \tag{22}$$

and

$$V_{t,i} = A'_{2,i} \, V_{u,i} \tag{23}$$

where $V_{L,i}$ are load element voltages for zone i, $V_{t,i}$ are voltages of non-load-non-generator elements (transmission elements) for zone i,

(18)

are all already

(19)

is completed.

(20)

.e $V_{E,i}$ for the

(21)

zone i can be
. Continuing
be obtained.
form (Fig. 4).

(22)

(23)

ıon-load-non-

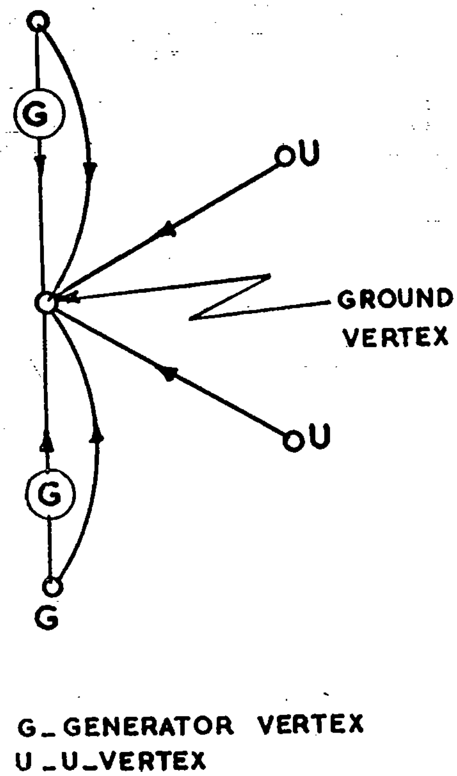


Fig. 3
G-U graph

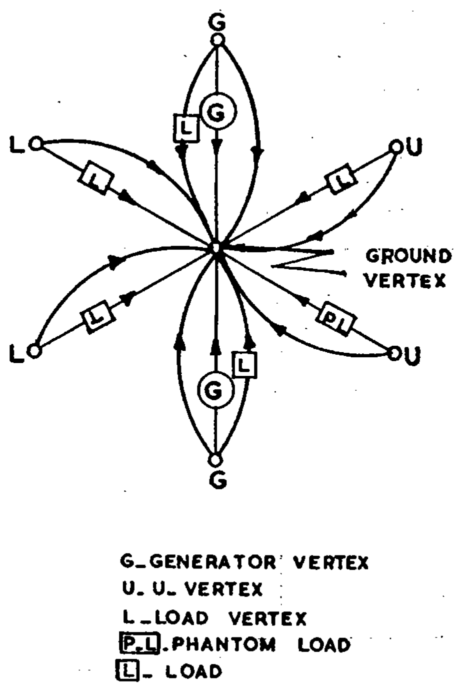


Fig. 4
All-vertex equivalent

$V_{u,i}$ are the node voltages for zone i, $A'_{1,i}$ is the transpose of the incidence matrix for the load elements in zone i, $A'_{2,1}$ is the transpose of the incidence matrix for the transmission elements of zone i.

With the calculation of equations (21) and (22), the complete restoration of voltages of all the elements of the original zone diagram is accomplished.

4. Example

The power system shown in Fig. 5 (which is the same as the one considered in a previous paper[2]) is used to illustrate the restoration technique.

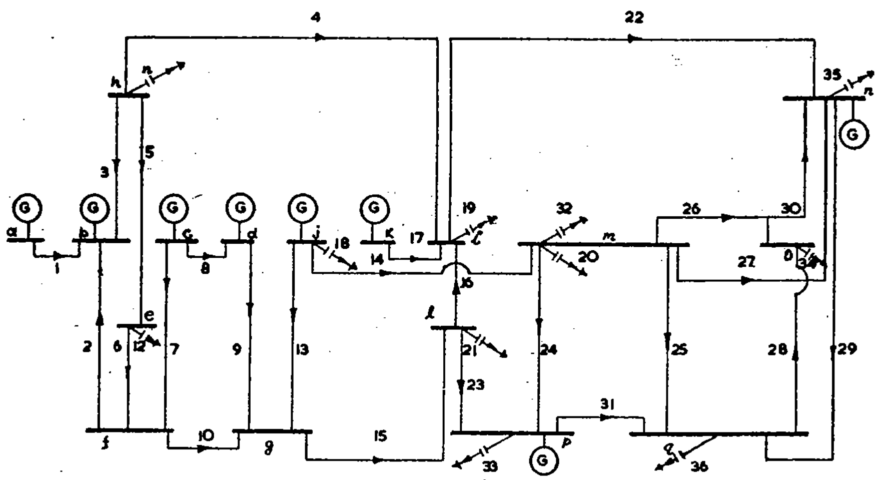


Fig. 5
Power system

First a set of voltages is assumed for the generator buses. This is given in Table 1.

Table 1
Assumed voltages for the generator buses

Generator bus		Voltage	
		Real part	Imaginary part
Zone 1	a	0.98	0
	b	0.99	0
	c	1.00	0
	d	1.01	0

Table I (*Contd.*)

Zone	2	j	1.02	0
		k	1.03	0
Zone	3	p	1.04	0
		n	1.05	0

The matrices M_1, M_2, M_3 and B^{-1} calculated during the reduction of the all zone G-U-graph shown in Fig. 7 to the generator daisy shown in Fig. 6 are given by equations (24), (25), (26) and (27) (only non-zero entries are given).

	Row	Column	Real part	Imaginary part	
	2	1	0.0	0.13828937E+01	
	2	2	0.0	0.46525300E+01	
$M_1 =$	3	1	0.0	0.10300973E+01	
	3	2	0.0	0.77021611	
	4	1	0.0	0.50505051E+01	(24)
	1	1	0.0	0.10638298E+02	
$M_2 =$	1	4	0.0	0.56818182E+01	
	2	2	0.0	0.416666667E+02	(25)
	1	3	0.0	0.40983607E+01	
$M_3 =$	1	4	0.0	0.43913998E+01	
	2	2	0.0	0.28571429E+01	
	2	4	0.0	0.32386704E+01	(26)
	1	1	0.0	0.4622325E—01	
	1	2	0.0	0.22924287E—02	
	1	3	0.0	0.12084284E—01	
	2	1	0.0	0.22924285E—02	
$B^{-1} =$	2	2	0.0	0.18388332E—01	
	2	3	0.0	0.10804562E—01	
	3	1	0.0	0.12084284E—01	
	3	2	0.0	0.10804562E—01	
	3	3	0.0	0.60234628E—01	
	4	4	0.0	0.75744328E—01	(27)

Substituting the matrices B^{-1}, M_1, M_2, M_3 and set of voltages assumed for the generator buses in equation. 18, the *U*-vertex voltages given in Table 2 are obtained.

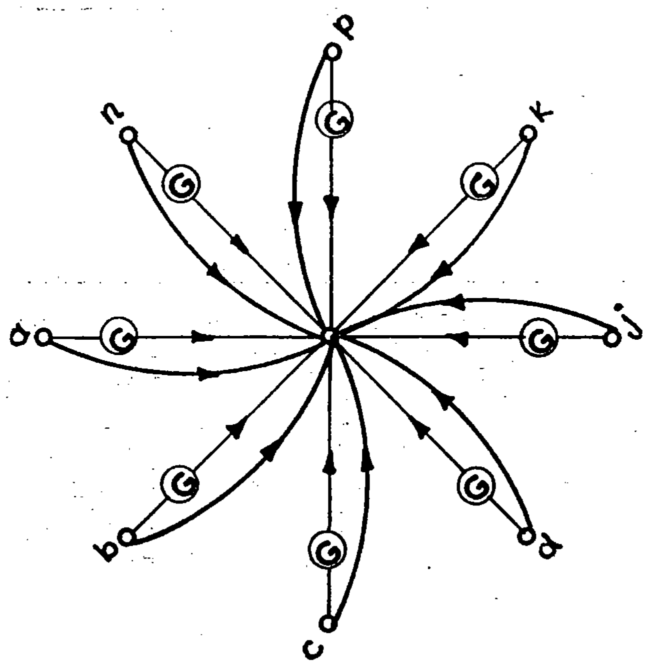


Fig. 6

System generator daisy

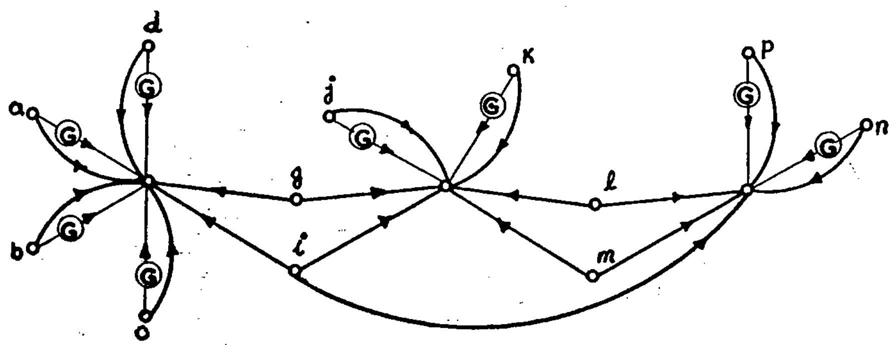


Fig. 7

All-zone G-U graph

Table 2

U-vertex voltages

U-vertex	Voltage	
	Real part	Imaginary part
g	1.0180628	0.0
i	1.0313118	0.0
l	1.0326964	0.0
m	1.0424779	0.0

With the computation of all the *U*-vertex voltages the voltages of all the buses of the all zone G-U-graph (Fig. 7) are obtained.

Next restoration of the voltages of the nodes of the all-vertex graph of each zone (Fig. 9) can be done from the node voltages of the corresponding G-U-graph (Fig. 8).

The all vertex graph of zone 2 is the same as its G-U-graph. Hence, the node voltages of the all vertex graph of zone 2 are the same as those of the corresponding G-U-graph.

The matrices $J_{EV,1}$, $J_{EV,3}$ calculated during the reduction of Figs. 9(i), 9(iii) to Figs. 8(i), 8(iii) are given by equations (28) and (29).

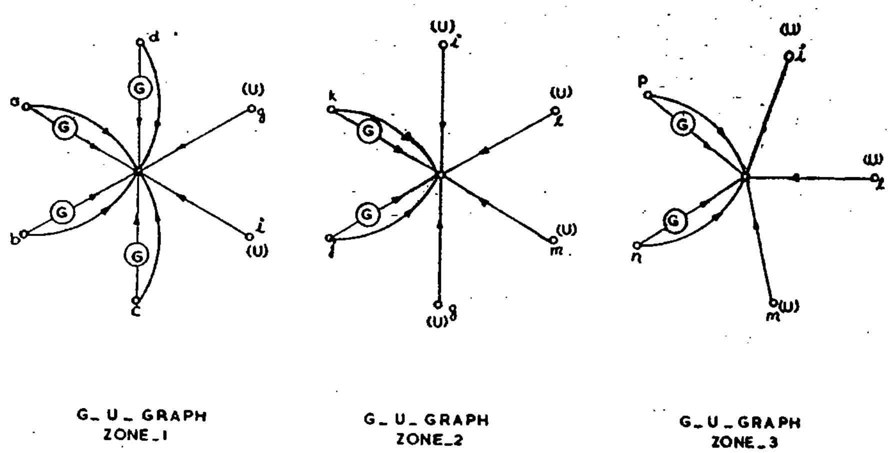
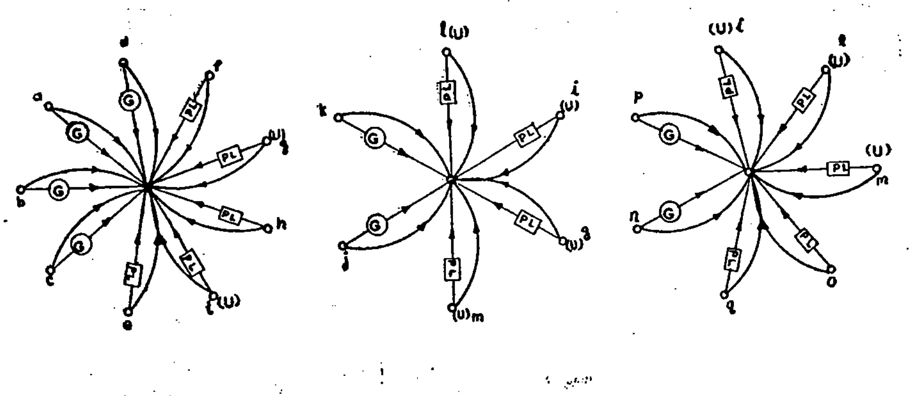


Fig. 8

G-U graphs



ALL-VERTEX EQUIVALENT ZONE-1 ALL-VERTEX EQUIVALENT ZONE-2 ALL-VERTEX EQUIVALENT ZONE-3

Fig. 9

All-vertex graphs

$J_{EV,1}$ =

Row	Column	Real part	Imaginary part
1	2	0.58687954	0.0
1	3	0.16574070	0.0
1	5	0.37245102E—01	0.0
1	6	0.22107755	0.0
2	2	0.49231017	0.0
2	3	0.36671463	0.0
2	5	0.82407782E—01	0.0
2	6	0.61617289E—01	0.0
3	2	0.61413396	0.0
3	3	0.10166853	0.0
3	5	0.22846860E—01	0.0
3	6	0.27037954	0.0

(28)

$J_{EV,3}$ =

1	1	0.15222929
1	2	0.58151981
1	5	0.27238387
2	1	0.69291649
2	2	0.19022835
2	5	0.11922156

(29)

The voltages of all the buses except the generator buses of the all vertex graph of each zone (1 and 3) can be obtained by substituting the corresponding J_{EV} matrix and the G-U-graph node voltages in equation (21).

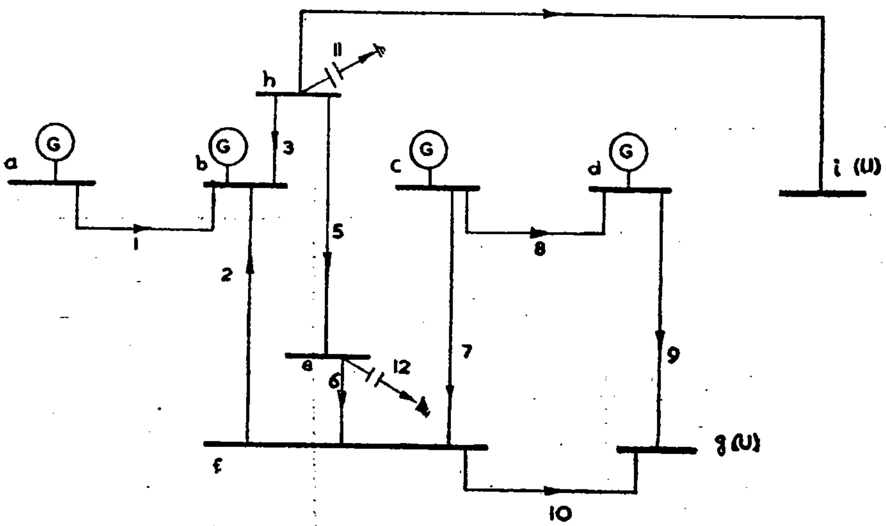
The voltages of all the buses in the power system of Fig. 5 are given in Table 3.

Table 3

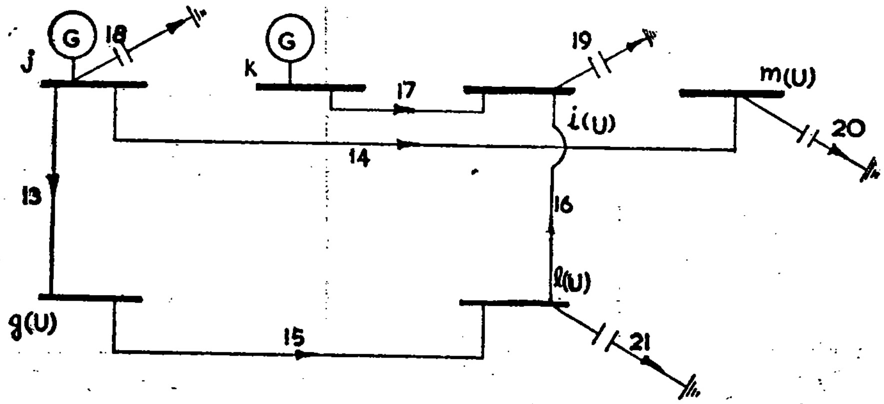
Voltages of the buses

Bus	Real part	Imaginary part
a	0.98	0
b	0.99	0
c	1.00	0
d	1.01	0
e	0.10126692E + 01	0
f	0.10015446E + 01	0
g	0.10180628E + 01	0
h	0.10117663E + 01	0
i	0.10313118E + 01	0
j	1.02	0
k	1.03	0
l	0.10326964E + 01	0
m	0.10424779E + 01	0
n	0.10500000E + 01	0
o	0.10528684E + 01	0
p	0.10400000E + 01	0
q	0.10446588E + 01	0

Using the transpose of the incidence matrix of the transmission elements in each zone and the corresponding node voltages, the voltages across all the transmission elements of zones 1, 2 and 3 shown in Fig. 10 can be calculated. These voltages and the admittances of transmission elements can be used to calculate the currents in the transmission elements. This completes the restroration process. The complete set of currents in all transmission elements is given in Table 4.

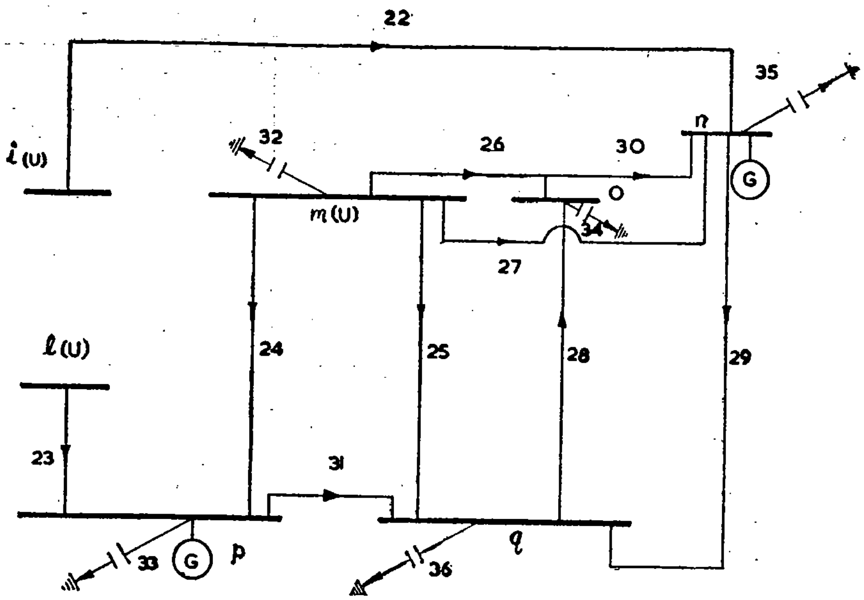


(i)
Zone—1



(ii)
Zone—2

Fig. 10 (*Contd.*)



(iii)
Zone—3
Fig. 10

Table 4
Currents in transmission elements

Transmission element	Current	
	Real part	Imaginary part
1	0	0.10000000E+01
2	0	— 0.14800769
3	0	— 0.31094714
4	0	0.14807197
5	0	0.31134483E—01
6	0	— 0.12091956
7	0	0.19307500E—01
8	0	0.45454545
9	0	0.40721213E—01

Table 4—(*contd.*)

10	0	0.46399439E—01
11	0	0.13174040
12	0	0.15205243
13	0	— 0.20608511E—01
14	0	0.12771534
15	0	0.66516363E—01
16	0	— 0.15050000E—01
17	0	0.54658334E—01
18	0	0.40800000E—01
19	0	0.13428539
20	0	0.41699116E—01
21	0	0.51634820E—01
22	0	0.53394858E—01
23	0	0.29932787E—01
24	0	—0.56061085E—02
25	0	0.56500001E—02
26	0	0.22786185E—01
27	0	0.11061911E—01
28	0	0.16065754E—01
29	0	—0.20543077E—01
30	0	—0.13790385E—01
31	0	0.83192858E—01
32	0	0.52123895E—01
33	0	0.52000000E—01
34	0	0.52643420E—01
35	0	0.14000000
36	0	0.52232940E—01

5. Acknowledgment

The authors are grateful to Dr. Myril B. Reed, formerly guest Professor, USAID, at the College of Engineering, Guindy and now Professor of Electrical Engineering, Colorado State University, Fort Collins, U.S.A. and to his wife Mrs. Georgia B. Reed for the course of lectures on 'Linear Graph Theory and Computer Programming' given at the College of Engineering, Guindy, under USAID programme.

The authors thank Shri T. Muthian, B.E., M.I.E., (India), Director of Technical Education, Madras and the authorities of the College of Engineering, Guindy, for the free use of the IBM 1620 digital computer at the Fundamental Engineering Research Establishment and the facilities at the College of Engineerign Guindy, Madras.

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