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ON THE NUMBER OF CONDUCTANCES REQUIRED FOR REALIZING Y AND K MATRICES

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SUMMARY

Upper bounds are established on the number of conductances required for realizing a real symmetric matrix **Y** as the short-circuit conductance matrix of a resistive n-port network containing no negative conductances, and for the realization of a real matrix **K** as the potential factor matrix of a similar network without negative conductances. These results are the consequence of the properties of the modified cut-set matrix of an n-port and a theorem in the theory of linear programming.

1. INTRODUCTION

Biorci[1,2] conjectured that, at most $n(n+1)/2$ conductances are required for realizing a real symmetric matrix as the short-circuit conductance matrix of a resistive n-port network containing no negative conductances. Even after several years of research, this conjecture has been neither proved nor disproved. However, a lower bound is known for the realization of **Y** matrices when the port configuration of the required network is specified.[3] In this paper, we establish upper bounds on the number of conductances required for realizing **Y** and **K** matrices. These results are the consequence of the properties of the modified cut-set matrix of an n-port and a theorem in the theory of linear programming.

2. AN UPPER BOUND ON THE NUMBER OF CONDUCTANCES REQUIRED FOR REALIZING A Y MATRIX

In this Section, we first summarize some results relating to the modified cut-set matrix of a resistive n-port network[4] and also state a theorem in the theory of linear programming. These results are then used to establish an upper bound on the number of conductances required for realizing an $(n\times n)$ **Y** matrix by an $(n+p)$-node n-port network.

Consider a resistive n-port network N. Let the port configuration T of N be in p connected parts $T_1, T_2, \ldots, T_p$. Permitting edges with zero conductances, the graph of N can be considered to be complete. Let T_0 be a tree of N such that $T\subseteq T_0$. The branches of T will be called the port branches, and the remaining branches of T_0 will be referred to as the non-port branches.

Let $\mathbf{C}_0$, the fundamental cut-set matrix of N with respect to the tree T_0 be partitioned as follows:

$$\mathbf{C}_0=\begin{bmatrix}\mathbf{C}_1\\ \hdashline \mathbf{C}_2\end{bmatrix} \tag{1}$$

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where the rows of $\mathbf{C}_1$ correspond to the port branches and those of $\mathbf{C}_2$ correspond to the non-port branches.

The cut-set admittance matrix $\mathbf{Y}_0$ of N with respect to the tree T_0 is defined as

$$\mathbf{Y}_0 = \mathbf{C}_0\mathbf{G}\mathbf{C}_0^t$$
$$= \left[\begin{array}{c|c} \mathbf{C}_1 \ \mathbf{G} \ \mathbf{C}_1^t & \mathbf{C}_1 \ \mathbf{G} \ \mathbf{C}_2^t \\ \hline \mathbf{C}_2 \ \mathbf{G} \ \mathbf{C}_1^t & \mathbf{C}_2 \ \mathbf{G} \ \mathbf{C}_2^t \end{array}\right] \quad \left[\begin{array}{c|c} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \hline \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{array}\right] \tag{2}$$

where $\mathbf{G}$ is the diagonal matrix of edge conductances of N. The short-circuit conductance matrix $\mathbf{Y}$ of N is given by

$$\mathbf{Y} = \mathbf{Y}_{11} - \mathbf{Y}_{12}\mathbf{Y}_{22}^{-1}\mathbf{Y}_{21} \tag{3}$$

The modified cut-set matrix[5] of N is defined as

$$\mathbf{C} = \mathbf{C}_1 - \mathbf{Y}_{12}\mathbf{Y}_{22}^{-1}\mathbf{C}_2 \tag{4}$$

The following results have been proved in Reference 4:

Theorem 1

Let $\mathbf{C}$ be the modified cut-set matrix of a connected resistive n-port network N having a port configuration T. Let $\mathbf{C}_0$ be the fundamental cut-set matrix of N with respect to a tree T_0 of which T is a subgraph. Further let $\mathbf{C}_1$ and $\mathbf{C}_2$, the submatrices of $\mathbf{C}_0$, correspond respectively to the port branches and the non-port branches of T_0. Let $\mathbf{Y}$ be the short-circuit conductance matrix of N with respect to the port configuration T.

(a) If $\mathbf{G}^k$ is the diagonal matrix of edge conductances of a connected n-port network N^* having the same port configuration as that of N and $\mathbf{C}\mathbf{G}^k\mathbf{C}_2^t = \mathbf{0}$, then the modified cut-set matrix of $\mathbf{N}^k$ is also equal to $\mathbf{C}$.

(b) Let

$$\mathbf{C}\mathbf{G}^k\mathbf{C}_1^t = \mathbf{y}$$

and

$$\mathbf{C}\mathbf{G}^k\mathbf{C}_2^t = \mathbf{0}$$

where $\mathbf{G}^k$ is the diagonal matrix of edge conductances of an n-port network N^* having the same port configuration as that of N. Then the modified cut-set matrix and the short-circuit conductance matrix of N^* are equal to $\mathbf{C}$ and $\mathbf{Y}$, respectively.

Theorem 2

Two n-port networks have the same modified cut-set matrix if they have the same $\mathbf{K}$ matrix.

Consider next the following set of m simultaneous equations in n variables $x_1, x_2, \ldots, x_n$:

$$\mathbf{AX} = \mathbf{b} \tag{5}$$

where $\mathbf{A}$ is an $(m \times n)$ real matrix, $\mathbf{X}$ is the column vector of the variables $x_1, x_2, \ldots, x_n$ and $\mathbf{b}$ is a column vector of real elements.

Any nonnegative solution of (5) is called a *feasible solution*. If any $(m \times m)$ nonsingular matrix is chosen from $\mathbf{A}$, and if all the $(n-m)$ columns of this matrix are set equal to zero, the solution to the resulting system of equations is called a *basic solution*. If a basic solution is feasible, then it is called a *basic feasible solution*. Thus the number of nonzero variables in a basic feasible solution will be less than or equal to m, the number of equations. The following result is proved in Reference 6.

Theorem 3

Consider a set of m simultaneous equations in n variables $(n \geq m)$

$$\mathbf{Ax} = \mathbf{b}$$

If there exists a feasible solution $\mathbf{x} \geqslant 0$ to these equations, then there exists a basic feasible solution.

We now prove the following theorem:

Theorem 4

If a matrix $\mathbf{Y}$ is realizable as the short-circuit conductance matrix of an $(n+p)$-node resistive n-port, then it can be realized by an n-port network containing at most $m = \{n(n+1)/2 + n(p-1)\}$ conductances.

Proof

Let the matrix $\mathbf{Y}$ be the short-circuit conductance matrix of an $(n+p)$-node n-port network contains m or less number of conductances, the theorem is proved. Otherwise, we proceed as follows to obtain an equivalent network containing, at most, m conductances.

Let $\mathbf{C}$ be the modified cut-set matrix of N_1. Let $\mathbf{C}_1$ and $\mathbf{C}_2$ be defined as in Theorem 1. Let $\mathbf{G}_1$ be the diagonal matrix of edge conductances of N_1.

Consider the following sets of equations:

$$\mathbf{CGC}_2^t = \mathbf{0} \tag{6a}$$

$$\mathbf{CGC}_1^t = \mathbf{Y} \tag{6b}$$

Note that each one of the matrices $\mathbf{C}$ and $\mathbf{C}_1$ has n rows and the matrix $\mathbf{C}_2$ has $(p-1)$ rows. Also the number of variables in $\mathbf{G}$ is equal to l where $l = (n+p)(n+p-1)/2$.

Hence, equation (6a) represents a set of $n(p-1)$ equations in l variables. Further, because of the symmetry of $\mathbf{Y}$, equation (6b) represents a set of $n(n+1)/2$ equations in l variables. Thus equations (6) represent a set of m equations in l variables.

The edge-conductance matrix $\mathbf{G}_1$ of the network N_1 is a feasible solution of (6). Hence, there exists a basic feasible solution $\mathbf{G}$. The number of nonzero variables in $\mathbf{G}_2$ is less than or equal to m. Since, by Theorem 1(b), $\mathbf{G}_2$ is the matrix of conductances of an n-port network N_2 whose short-circuit conductance matrix is equal to $\mathbf{Y}$, we conclude that, for the given matrix $\mathbf{Y}$, there exists an $(n+p)$-node realization containing, at most, m conductances.

Example 1

The matrix $\mathbf{Y}$ given below is the short-circuit conductance matrix of a 3-port network N_1 having the port configuration T shown in Figure 1.

$$\mathbf{Y} = \begin{bmatrix} 1{\cdot}00 & -0{\cdot}08 & -0{\cdot}08 \\ -0{\cdot}08 & 2{\cdot}00 & 0{\cdot}08 \\ -0{\cdot}08 & 0{\cdot}08 & 3{\cdot}00 \end{bmatrix}$$

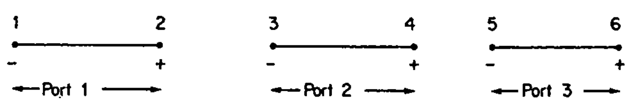

Figure 1. Port configuration for Example 1

The diagonal matrix $\mathbf{G}_1$ of edge conductances (all in siemens) of N_1 is given by

$$\begin{aligned} \mathbf{G}_1 = \text{diag}\{ & g_{12} \quad g_{13} \quad g_{14} \quad g_{15} \quad g_{16} \quad g_{23} \quad g_{24} \quad g_{25} \\ & g_{26} \quad g_{34} \quad g_{35} \quad g_{36} \quad g_{45} \quad g_{46} \quad g_{56}\} \\ = \text{diag}\{ & 0{\cdot}49 \quad 0{\cdot}06 \quad 0{\cdot}14 \quad 0{\cdot}45 \quad 0{\cdot}05 \quad 0{\cdot}54 \quad 1{\cdot}26 \quad 0{\cdot}45 \\ & 0{\cdot}05 \quad 1{\cdot}08 \quad 0{\cdot}70 \quad 0{\cdot}70 \quad 0{\cdot}30 \quad 0{\cdot}30 \quad 2{\cdot}33\} \end{aligned}$$

The modified cut-set matrix $\mathbf{C}$ of N_1 is obtained as follows:

$$\begin{array}{c} \\ \mathbf{C}= \end{array}\begin{array}{ccccccccccccccc} g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{23} & g_{24} & g_{25} & g_{26} & g_{34} & g_{35} & g_{36} & g_{45} & g_{46} & g_{56} \end{array}$$

$$\mathbf{C}=\begin{bmatrix} 1 & 0.8 & 0.8 & 0.7 & 0.7 & -0.2 & -0.2 & -0.3 & -0.3 & 0 & -0.1 & -0.1 & -0.1 & -0.1 & 0 \\ 0 & -0.6 & 0.4 & -0.2 & -0.2 & -0.6 & 0.4 & -0.2 & -0.2 & 1 & 0.4 & 0.4 & -0.6 & -0.6 & 0 \\ 0 & 0.1 & 0.1 & -0.3 & 0.7 & 0.1 & 0.1 & -0.3 & 0.7 & 0 & -0.4 & 0.6 & -0.4 & 0.6 & 1 \end{bmatrix}$$

Choosing the edges e_{23} and e_{45} as the nonport branches, we obtain the matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ as follows:

$$\begin{array}{ccccccccccccccc} g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{23} & g_{24} & g_{25} & g_{26} & g_{34} & g_{35} & g_{36} & g_{45} & g_{46} & g_{56} \end{array}$$

$$C_1=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{C}_2=\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

A basic feasible solution $\mathbf{G}_2$ for the set of equations

$$\mathbf{CGC}_1^t=\mathbf{Y}$$

and

$$\mathbf{CGC}_2^t=\mathbf{0}$$

is then obtained using the MPS package available with the IBM 370/155 computer system.

The nonzero entries of $\mathbf{G}_2$ are as follows:

$$\begin{array}{llll} g_{12}=0.64800 & g_{15}=0.32000 & g_{25}=0.42286 & g_{35}=0.17143 \\ g_{13}=0.08000 & g_{23}=0.14857 & g_{26}=0.17143 & g_{36}=0.28571 \\ g_{14}=0.08000 & g_{24}=0.72000 & g_{34}=1.68000 & g_{56}=2.70857 \end{array}$$

For the case under consideration, $n=3$ and $p=3$, and so $m=12$. Note that the number of nonzero entries in $\mathbf{G}_2$ is equal to 12. Thus the 3-port network N_2 of which $\mathbf{G}_2$ is the matrix of edge conductances is a realization of the given matrix $\mathbf{Y}$ containing, at most, m conductances.

3. AN UPPER BOUND ON THE NUMBER OF CONDUCTANCES REQUIRED FOR REALIZING A K MATRIX

In this Section, we establish an upper bound on the number of conductances required for the realization of a real matrix $\mathbf{K}$ as the potential factor matrix of an $(n+p)$-node n-port resistive network containing no negative conductances.

Theorem 5

If a real matrix $\mathbf{K}$ is realizable as the potential factor matrix of an $(n+p)$-node n-port network then it can be realized by an n-port network containing, at most, $\{n(p-1)+(p-1)\}$ conductances.

Proof

Let the given matrix $\mathbf{K}$ be the potential factor matrix of an $(n+p)$-node n-port network N_1. If N_1 contains $\{n(p-1)+(p-1)\}$ or less conductances, the theorem is proved. Otherwise, we proceed as follows to obtain an equivalent n-port network N_2 containing, at most, $\{n(p-1)+(p-1)\}$ conductances.

Let $\mathbf{C}$ be the modified cut-set matrix of the n-port network N_1 realizing the given $\mathbf{K}$ matrix. Let $\mathbf{G}_1$ be the diagonal matrix of edge conductances of N_1. Let the matrix $\mathbf{C}_2$ be defined as in Theorem 1.

Consider any diagonal matrix $\mathbf{G}_2$ of real nonnegative entries satisfying the equation

$$\mathbf{C}\mathbf{G}_2\mathbf{G}_2^t=\mathbf{0} \tag{7}$$

Let $\mathbf{G}_2$ be the matrix of edge conductances of a connected $(n+p)$-node n-port network N_2. Then, by Theorem 1a, the modified cut-set matrix of N_2 is equal to $\mathbf{C}$. Also, by Theorem 2, the potential factor matrix of N_2 is equal to the matrix $\mathbf{K}$. To ensure that a solution $\mathbf{G}_2$ of (7) corresponds to a connected n-port network, we proceed as follows:

Let the p connected parts of the port configuration of N_1 be denoted by $T_1, T_2, \ldots, T_p$. Let $(S_{ij})_1$ denote the sum of the conductances in the given network N_1 connecting vertices in T_i to those in T_j. $(S_{ij})_2$ will refer to the corresponding quantity in the required network N_2. Note that the port configuration of N_2 will be the same as that of N_1.

If all the ports of N_2 are short-circuited, the network $(N_2)_S$ that results will have p vertices. $(S_{ij})_2$s will represent the different conductances of $(N_2)_S$. If $(N_2)_S$ is connected, N_2 will also be connected.

Choose a set of $(p-1)$ positive conductances $(S_{ij})_1$s such that they constitute a tree of $(N_1)_S$. Let these conductances be denoted by

$$(S_{i_1k_1})_1,\ (S_{i_2k_2})_1,\ \ldots,\ (S_{i_{p-1}k_{p-1}})_1$$

If the corresponding conductances of $(N_2)_S$ are also positive, then, as mentioned earlier, the n-port network N_2 will be connected.

Consider then the following set of $(p-1)$ equations:

$$(S_{i_jk_j})=(S_{i_jk_j})_1 \qquad j=1,2,\ldots,p-1 \tag{8}$$

Note that each $(S_{i_jk_j})$ can be written as a sum of the entries of the matrix $\mathbf{G}$.

Any solution of (7) and (8) will correspond to the diagonal matrix of edge conductances of a connected n-port network.

Equations (7) and (8) together represent a set of $\{n(p-1)+(p-1)\}$ equations in $(n+p)(n+p-1)/2$ variables. $\mathbf{G}_1$, the diagonal matrix of edge conductances of N_1, is a feasible solution of these equations. Hence a basic feasible solution $\mathbf{G}_2$ exists. The number of nonzero conductances in this basic feasible solution is less than or equal to $\{n(p-1)+(p-1)\}$. Thus there exists a network N_2 (of which $\mathbf{G}_2$ is the diagonal matrix of edge conductances) containing, at most, $\{n(p-1)+(p-1)\}$ conductances. As stated earlier, the network N_2 will realize the given matrix $\mathbf{K}$. Hence the theorem.

Example 2

The matrix $\mathbf{K}$ given below is the potential factor matrix of a 4-port network N_1 having the port configuration shown in Figure 2.

$$\mathbf{K}=\begin{bmatrix} 1 & 1 & 1 & \frac{7}{9} \\ 0 & 1 & 1 & \frac{5}{9} \\ 0 & 0 & 1 & \frac{3}{9} \\ \frac{5}{9} & \frac{5}{9} & \frac{5}{9} & 1 \end{bmatrix}$$

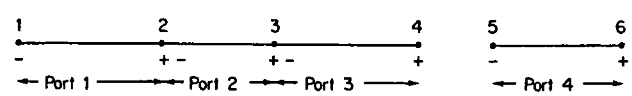

Figure 2. Port configuration for Example 2

The matrix $\mathbf{G}_1$ of edge conductances (all in siemens) of N_1 is given by

$$\mathbf{G}_1 = \operatorname{diag}\{g_{12}\ g_{13}\ g_{14}\ g_{15}\ g_{16}\ g_{23}\ g_{24}\ g_{25}\ g_{26}\ g_{34}\ g_{35}\ g_{36}\ g_{45}\ g_{46}\ g_{56}\}$$
$$= \operatorname{diag}\{\tfrac{5}{9}\ \tfrac{4}{9}\ \tfrac{2}{9}\ \tfrac{8}{9}\ \tfrac{10}{9}\ \tfrac{6}{9}\ \tfrac{3}{9}\ \tfrac{8}{9}\ \tfrac{10}{9}\ \tfrac{7}{9}\ \tfrac{8}{9}\ \tfrac{10}{9}\ \tfrac{12}{9}\ \tfrac{15}{9}\ \tfrac{8}{9}\}$$

The modified cut-set matrix $\mathbf{C}$ of N_1 is obtained as follows:

$$\begin{array}{c} \\ \mathbf{C}= \end{array}\begin{array}{c} \begin{matrix} g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{23} & g_{24} & g_{25} & g_{26} & g_{34} & g_{35} & g_{36} & g_{45} & g_{46} & g_{56} \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & \frac{7}{9} & \frac{7}{9} & 0 & 0 & -\frac{2}{9} & -\frac{2}{9} & 0 & -\frac{2}{9} & -\frac{2}{9} & -\frac{2}{9} & -\frac{2}{9} & 0 \\ 0 & 1 & 1 & \frac{5}{9} & \frac{5}{9} & 1 & 1 & \frac{5}{9} & \frac{5}{9} & 0 & -\frac{4}{9} & -\frac{4}{9} & -\frac{4}{9} & -\frac{4}{9} & 0 \\ 0 & 0 & 1 & \frac{3}{9} & \frac{3}{9} & 0 & 1 & \frac{3}{9} & \frac{3}{9} & 1 & \frac{3}{9} & \frac{3}{9} & -\frac{6}{9} & -\frac{6}{9} & 0 \\ 0 & 0 & 0 & -\frac{5}{9} & \frac{4}{9} & 0 & 0 & -\frac{5}{9} & \frac{4}{9} & 0 & -\frac{5}{9} & \frac{4}{9} & -\frac{5}{9} & \frac{4}{9} & 1 \end{bmatrix} \end{array}$$

Choosing the edge e_{45} connecting the vertices 4 and 5 as the nonport branch we obtain $\mathbf{G}_2$ as follows:

$$\begin{array}{c} \begin{matrix} g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{23} & g_{24} & g_{25} & g_{26} & g_{34} & g_{35} & g_{36} & g_{45} & g_{46} & g_{56} \end{matrix} \\ \mathbf{C}_2 = [\begin{matrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{matrix}] \end{array}$$

In $(N_1)_S$, S_{12}, the combination of the conductances $g_{15}, g_{16}, g_{25}, g_{26}, g_{35}, g_{36}, g_{45}$ and g_{46} forms a tree. A basic feasible solution $\mathbf{G}_2$ to the following sets of equations is required.

$$\mathbf{C}\mathbf{G}_2\mathbf{G}_2^t = \mathbf{0}$$

$$(S_{12}) = (S_{12})_1 \qquad \text{i.e.} = 9$$

After substituting for $\mathbf{C}$ and $\mathbf{C}_2$, the above simplifies to the following:

$$\begin{bmatrix} 0 & 0 & 0 & 7 & 7 & 0 & 0 & -2 & -2 & 0 & -2 & -2 & -2 & -2 & 0 \\ 0 & 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 & 0 & -4 & -4 & -4 & -4 & 0 \\ 0 & 0 & 0 & 3 & 3 & 0 & 0 & 3 & 3 & 0 & 3 & 3 & -6 & -6 & 0 \\ 0 & 0 & 0 & -5 & 4 & 0 & 0 & -5 & 4 & 0 & -5 & 4 & -5 & 4 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} g_{12} \\ g_{13} \\ g_{14} \\ g_{15} \\ g_{16} \\ g_{23} \\ g_{24} \\ g_{25} \\ g_{26} \\ g_{34} \\ g_{35} \\ g_{36} \\ g_{45} \\ g_{46} \\ g_{56} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 9 \end{bmatrix}$$

Using the MPS package, the following basic feasible solution $\mathbf{G}_2$ is obtained. The nonzero entries of $\mathbf{G}_2$ (all in siemens) are given by

$$g_{16} = 2{\cdot}0 \qquad g_{25} = 2{\cdot}0 \qquad g_{36} = 2{\cdot}0 \qquad g_{45} = 2{\cdot}0 \qquad g_{46} = 1{\cdot}0$$

Note that, in this case, $n = 4$ and $p = 2$. Hence $\{n(p-1)+(p-1)\} = 5$. It may be seen that $\mathbf{G}_2$ contains five nonzero entries. The network N_2 of which $\mathbf{G}_2$ is the diagonal matrix of edge conductances is a realization of the matrix **K** containing $\{n(p-1)+(p-1)\}$ conductances.

4. CONCLUSIONS

In this paper, we have established upper bounds on the number of conductances required for realizing **Y** and **K** matrices. According to Theorem 4, the maximum number of conductances required for realizing any $(n \times n)$ **Y** matrix by an $(n+2)$-node n-port network is equal to $\{n(n+1)/2+n\}$. In a recent paper, it was shown that any **Y** matrix realizable by an $(n+1)$-node n-port network containing no zero conductances can be realized by an n-port network containing, at most, $\{n(n+1)/2+1\}$ conductances, which is less than the maximum number of conductances required according to Theorem 5. It may, therefore, be expected that the approach of Reference 7 can be generalized to obtain $(n+p)$-node realizations of **Y** matrices of $(n+1)$-node n-port networks containing, at most, $\{n(n+1)/2(p-1)/2\}$ conductances.

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