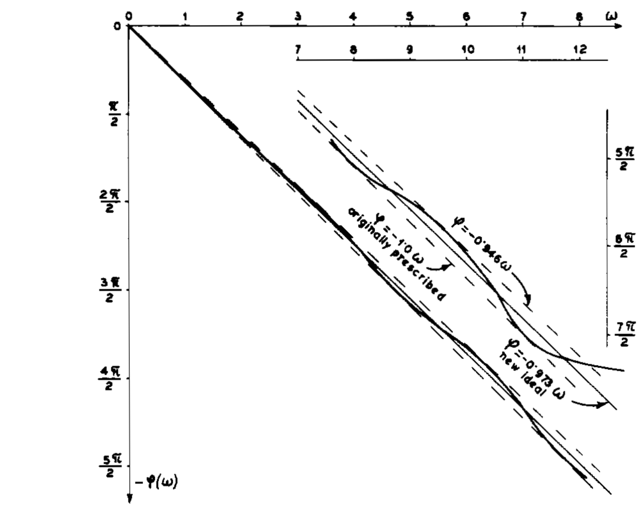

Fig. 1.

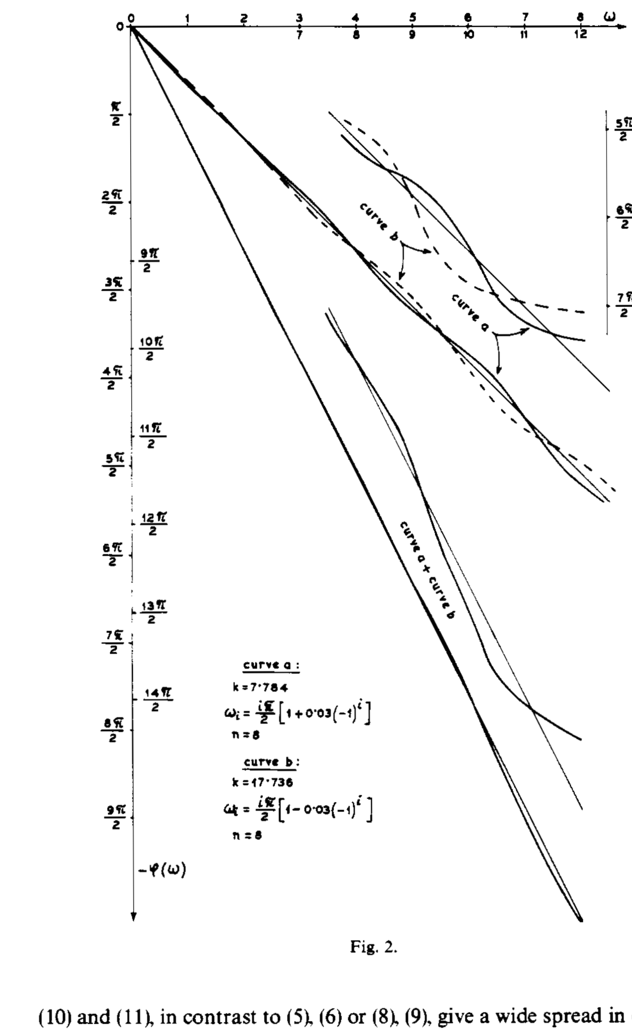

Fig. 2.

(10) and (11), in contrast to (5), (6) or (8), (9), give a wide spread in constant k. For the example at hand the following values for constant k have been found:

Curve a:

7.750; 7.784; 7.574; 7.985; 7.000; 8.290; 5.763; 7.957.

Curve b:

17.956; 17.736; 16.544; 15.782; 12.586; 11.472; 6.830; 3.412.

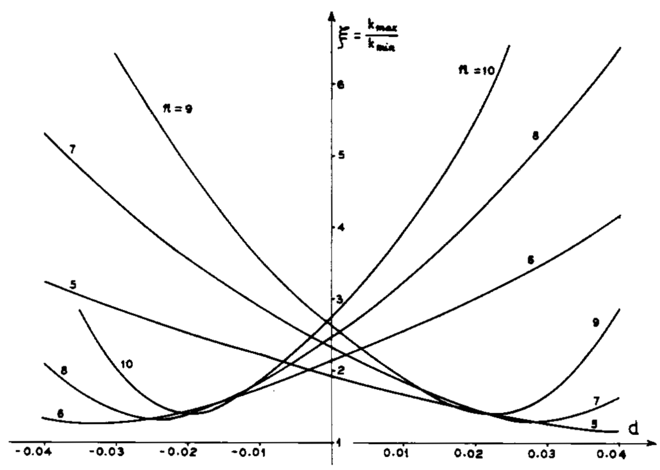

Fig. 3.

It is the large variation in constant k for curve b which prevents the cancellation of all the ripples.

If we designate k_{max} and k_{min} as the maximum and minimum values obtained for constant k, we can define the *constant k spread*

$$\xi = \frac{k_{max}}{k_{min}}$$

which may be used as an approximate measure of the quality of the linear phase approximation. For the example at hand, curve a with $\xi = 1.44$ is a considerably better approximation than curve b with $\xi = 5.26$. Instead of using four relations (8) to (11) to define the zeros, we can use only one, and allow d to be a positive or negative real number. Now if we prescribe different values for d and calculate the resulting ξ, we obtain the interesting curves in Fig. 3. It seems that the best value for displacement d is somewhere between 0.2 and 0.3 (or -0.2 and -0.3). The constant k spread is quite low (1.3 to 1.4) in that range, so the phase angle curve could be easily maintained within the limits of ± 2 to ± 3 percent.

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Pseudo-Series Combination of *n*-Port Networks

***Abstract*—The "pseudo-series" combination of two *n*-port networks is defined. A necessary and sufficient condition is given for the combined *n*-port network to have an open-circuit impedance matrix equal to the sum of the corresponding matrices of the component networks.**

A problem that is of interest in the synthesis of n-port networks without transformers may be stated as follows. Given an open-circuit impedance matrix Z, what are the conditions under which Z may be expressed as the sum of $Z_a, Z_b, \cdots, Z_m$ such that each of the component matrices is conveniently realized by an n-port network, and such that a suitable combination of the component networks realizes the given Z-matrix? In this letter we define the pseudo-series combination of n-port networks[1] and give a necessary and sufficient condition for the Z-matrix of the combined network to be equal to the sum of the Z-matrices of the component networks.

We consider two connected networks N_a and N_b having only RLC ele-

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ments and identical edge and port configurations. Let Z_a and Z_b be the open-circuit impedance matrices of N_a and N_b, respectively. From N_a and N_b we form a third n-port network N_c also having the same edge and port configurations and orientations, but having the impedance of each edge as the sum of the impedances of the corresponding edges of N_a and N_b. Then N_c is said to be the *pseudo-series combination* of N_a and N_b.[1] If the open-circuit impedance matrix Z_c of N_c is equal to Z_a+Z_b, then we qualify N_c as the *proper pseudo-series combination* of N_a and N_b.

Let Z_{ea} and Z_{eb} be the diagonal edge impedance matrices of the networks N_a and N_b. Let $B=[B_1/B_2]$ be the common fundamental circuit matrix of N_a and N_b with respect to a tree which is so chosen that all the ports are included in a cotree, and let the rows of the submatrix B_1 correspond to the port chords and those of B_2 to the nonport chords. Then we have the following as the loop-impedance matrices of N_a and N_b:

$$\tilde{Z}_a = \left[\frac{B_1}{B_2}\right] Z_{ea}[B_1'|B_2'] = \left[\begin{array}{c|c} B_1Z_{ea}B_1' & B_1Z_{ea}B_2' \\ \hline B_2Z_{ea}B_1' & B_2Z_{ea}B_2' \end{array}\right] = \left[\begin{array}{c|c} Z_{11a} & Z_{12a} \\ \hline Z_{21a} & Z_{22a} \end{array}\right] \quad (1)$$

$$\tilde{Z}_b = \left[\frac{B_1}{B_2}\right] Z_{eb}[B_1'|B_2'] = \left[\begin{array}{c|c} B_1Z_{eb}B_1' & B_1Z_{eb}B_2' \\ \hline B_2Z_{eb}B_1' & B_2Z_{eb}B_2' \end{array}\right] = \left[\begin{array}{c|c} Z_{11b} & Z_{12b} \\ \hline Z_{21b} & Z_{22b} \end{array}\right]. \quad (2)$$

Assuming that the Z_{22} matrices are nonsingular, the modified circuit matrices B_a and B_b for the networks N_a and N_b as defined by Cederbaum[2] are given by

$$B_a = B_1 - Z_{12a}Z_{22a}^{-1}B_2 \quad (3)$$

$$B_b = B_1 - Z_{12b}Z_{22b}^{-1}B_2. \quad (4)$$

It can readily be shown that the pseudo-series combination of N_a and N_b is proper if their modified circuit matrices B_a and B_b are equal.[1] In what follows we show that the equality of the modified circuit matrices B_a and B_b is also a necessary condition in the general case for the proper pseudo-series combination of N_a and N_b.

Let the pseudo-series combination of N_a and N_b be proper, yielding the combined network N_c. We then have the following relations, where the vectors I_p and V_p refer to the port currents and voltages and the vector I_n refers to the currents in the nonport chords.

Network N_a:

$$V_{pa} = Z_{11a}I_p + Z_{12a}I_{na} \quad (5)$$

$$0 = Z_{21a}I_p + Z_{22a}I_{na}. \quad (6)$$

Network N_b:

$$V_{pb} = Z_{11b}I_p + Z_{12b}I_{nb} \quad (7)$$

$$0 = Z_{21b}I_p + Z_{22b}I_{nb}. \quad (8)$$

Network N_c:

$$V_{pc} = (Z_{11a} + Z_{11b})I_p + (Z_{12a} + Z_{12b})I_{nc} \quad (9)$$

$$0 = (Z_{21a} + Z_{21b})I_p + (Z_{22a} + Z_{22b})I_{nc}. \quad (10)$$

In the foregoing, the matrices Z_{11a}, Z_{11b}, Z_{22a}, and Z_{22b} are symmetrical, and the matrices Z_{12a} and Z_{12b} are the transposes of the matrices Z_{21a} and Z_{21b}, respectively.

Since N_c is the proper pseudo-series combination of N_a and N_b, we have $Z_c=Z_a+Z_b$. For any arbitrary port current vector I_p we then have

$$V_{pc} = V_{pa} + V_{pb}, \quad (11)$$

leading to the following relation:

$$Z_{12a}I_{na} + Z_{12b}I_{nb} = (Z_{12a} + Z_{12b})I_{nc}. \quad (12)$$

Also, from (6), (8), and (10) we have

$$Z_{22a}I_{na} + Z_{22b}I_{nb} = (Z_{22a} + Z_{22b})I_{nc}. \quad (13)$$

Premultiplying the terms in (12) by I_p' (the transpose of I_p) and using (6) and (8), we obtain

$$I_{na}'Z_{22a}(I_{na} - I_{nc}) + I_{nb}'Z_{22b}(I_{nb} - I_{nc}) = 0. \quad (14)$$

Premultiplying the terms in (13) by I_{nc}', we obtain

$$I_{nc}'Z_{22a}(I_{na} - I_{nc}) + I_{nc}'Z_{22b}(I_{nb} - I_{nc}) = 0. \quad (15)$$

From (14) and (15) we get

$$(I_{na} - I_{nc})'Z_{22a}(I_{na} - I_{nc}) + (I_{nb} - I_{nc})'Z_{22b}(I_{nb} - I_{nc}) = 0. \quad (16)$$

If the two matrices Z_{22a} and Z_{22b} are positive definite for positive real values of the complex frequency variable s, the only way in which (16) is satisfied is when both terms on the left-hand side are zero, i.e.,

$$I_{na} = I_{nb} = I_{nc}. \quad (17)$$

This leads to

$$Z_{22a}^{-1}Z_{21a}I_p = Z_{22b}^{-1}Z_{21b}I_p = (Z_{22a} + Z_{22b})^{-1}(Z_{21a} + Z_{21b})I_p. \quad (18)$$

Since this is to be valid for all I_p, it follows that

$$Z_{12a}Z_{22a}^{-1} = Z_{12b}Z_{22b}^{-1} = (Z_{12a} + Z_{12b})(Z_{22a} + Z_{22b})^{-1}. \quad (19)$$

Equation (19) is true not only for real positive values of s, but for all values of s by virtue of analytical continuation property. From (3), (4), and (19), it follows that the modified circuit matrices B_a and B_b of N_a and N_b are the same. It is also readily verified that N_c also has the same modified circuit matrix.

The foregoing discussion leads to the following theorem.

Theorem

The pseudo-series combination of two n-port networks N_a and N_b having nonsingular Z_{22} matrices that are positive definite for real positive values of the complex frequency variable s is proper if and only if the modified circuit matrices of N_a and N_b are equal.

It is well known that the principal minors of the loop-impedance matrices of RLC networks containing only positive resistances, inductances, and capacitances are positive definite or positive semidefinite for real positive values of s. For the Z-matrix to exist, however, the Z_{22} matrices should be nonsingular and hence positive definite for real positive values of s. Hence the criterion contained in the theorem is generally applicable to such networks.

The extension of the foregoing result to more than two n-port networks is obvious. A straightforward application of this criterion to test for the proper pseudo-series combination of p networks requires the inversion of p Z_{22} matrices. It can be shown, however, that the inversion of one such matrix will do for this purpose. A convenient test procedure incorporating this criterion is given elsewhere.[3]

Lempel and Cederbaum[4] have recently given a similar necessary and sufficient condition in terms of the modified cut-set matrix for the proper parallel interconnection of n-port networks without internal vertices. It can be shown that if one considers the pseudo-parallel combination instead of the regular parallel interconnection (the internal vertices are also interconnected in the pseudo-parallel combination), then the same criterion is also valid for n-port networks with internal vertices.

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