track of whether a given port variable is currently assumed to represent a current or a voltage.

III. Conclusions

Given a set of state equations representing an electrical network, it is intuitively clear that synthesis will be simple provided that all elements of the state vector represent inductor currents or capacitor voltages. If this condition does not hold, it is desirable to find a transformation of the state equations such that the new state vector satisfies the condition. Such a transformation may easily be found [1] when the solution P of (3) is known; in this paper, the transformation has in effect been found without solution of (3). It is apparent that the methods of this paper work *precisely when (3) is difficult to solve.*

The techniques discussed here may actually be used to solve these equations. To see this, note that step 4) of the algorithm uses a change of variables of the form $\hat{x} = Tx$; it is easily shown that this induces a corresponding change $P = T'\hat{P}T$ in (3). A straightforward calculation then shows that $\hat{P}$ is block diagonal, with one of the blocks a known quantity. Each time step 4) is applied, a further block of P becomes known. When $(J + J')$ ultimately becomes nonsingular, the remaining unknown components of P can be found by solving a reduced-dimensional form of (3).

It is of some interest to note that the synthesis procedure leaves open a number of options. To see this, suppose that $Z(s)$ is the immittance to be synthesized, and that $Z_1(s)$ is the immittance obtained (with $Z_1(\infty)$ nonsingular and symmetric) after the lossless extractions of Section II. Then the following may easily be verified.

1) A synthesis of $Z(s)$ with the minimum possible number of reactive elements follows from a synthesis of $Z_1(s)$ with the minimum possible number of reactive elements.

2) A synthesis of $Z(s)$ with the minimum possible number of resistors follows from a synthesis of $Z_1(s)$ with the minimum possible number of resistors.

3) If $Z(s)$ is symmetric, then so is $Z_1(s)$, and a reciprocal synthesis of $Z(s)$ follows from a reciprocal synthesis of $Z_1(s)$.

In consequence, the procedures of this paper may be used as a preamble to minimal-reactive, minimal-resistive, or reciprocal syntheses, without prejudice to the aims of these syntheses.

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Active *RC* *n*-Port Network Synthesis Using Nullators and Norators

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***Abstract*—A new method of synthesizing active *RC* *n*-port networks using nullators and norators is given. The method based on the reactance extraction principle uses a minimum number of grounded capacitors and gains its importance from the fact that the synthesis procedure does not depend on any topological considerations. A bound on the maximum number of nullator-norator pairs required to realize any arbitrary $Y(s)$ matrix is obtained.**

Manuscript received February 5, 1973; revised May 10, 1973. This work is part of a dissertation submitted to the Indian Institute of Technology, Madras, India, in partial fulfillment of the requirements for the Master of Science degree, by P. A. Ramamoorthy.
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I. Introduction

The USE of nullator-norator pairs as the active building blocks in active *RC* network synthesis is well known. As an ideal transistor and an operational amplifier can be approximated by a nullator-norator pair [1], much interest has been shown in analyzing [1]-[4] and synthesizing [5]-[8] with nullators and norators as a prelude to the analysis and synthesis of networks containing resistors, capacitors, and transistors or operational amplifiers. In a recent paper, Yarlagadda and Ye [9] present a method of synthesizing networks with resistors, capacitors, and nullator-norator pairs which heavily depends on the network topology. Here an entirely different approach is adopted, and a simple and general network realization procedure is established; only ordinary matrix manipulations are needed. For a given $Y(s)$ several realizations are shown to exist and for a certain choice of parameters, a structure similar to that of Yarlagadda and Ye [9] is obtained. An upper bound on the maximum number of nullator-norator pairs required is obtained and is found to be less than what the method due to Yarlagadda and Ye [9] suggests.

The synthesis method presented here is based on well-known state-space techniques and the reactance extraction principle. The use of state-space techniques in the synthesis of passive [10]-[12] and active networks [13]-[16] is attractive, since it can lead to a network employing the minimum number of reactive elements, all with one end grounded, a favorable feature for integrated circuit fabrication [17].

II. Realization Procedure

We restrict our attention for a given short-circuit admittance matrix $Y(s)$ to the synthesis of networks with resistors, capacitors, and nullator-norator pairs. Synthesis for other types of network descriptions, such as a voltage gain matrix, can in general be related to the synthesis of a $Y(s)$ [15].

The given $n \times n$ matrix $Y(s)$ of real rational functions of the complex frequency variable s is assumed to be regular at infinity. If $Y(s)$ is not regular at infinity, it can be transformed to one regular at infinity by a Möbius transformation [18]. The synthesis procedure to be discussed is then applied to the newly formed matrix. The realization for the original matrix can then be obtained by the inverse transformation [14]. Thus the assumption that $Y(s)$ is regular at $s = \infty$ involves no loss in generality.

Following the steps of Melvin and Bickart [14], the problem of synthesizing an active RC network from its short-circuit admittance representation reduces to that of synthesizing an $(n+p)$-port network N_R of resistors and nullator-norator pairs, loaded by a subnetwork N_c containing p star-connected capacitors—where p equals the degree of the matrix $Y(s)$ [19]—as in Fig. 1, from its short-circuit conductance matrix G_{NR}. The matrix G_{NR} is obtained as

$$G_{NR} = \begin{bmatrix} J & H \\ -CG & -CF \end{bmatrix} \tag{1}$$

where C is a diagonal matrix with the capacitor values of N_c being the diagonal entries. We note that

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_{NR} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{2}$$

where e_1 and i_1 are n-vectors of input voltages and currents, respectively, and e_2 and i_2 are p-vectors of capacitor voltages and currents of the subnetwork N_c.

The $(n+p)$-port network N_R may be viewed as a $(3n + 3p + 2a)$-port resistor network N_R' with nullator-norator pairs connected to the other $2(n + p + a)$-ports, as shown in Fig. 2. The value of a will be specified later. We shall denote the port voltage vector E of N_R' by

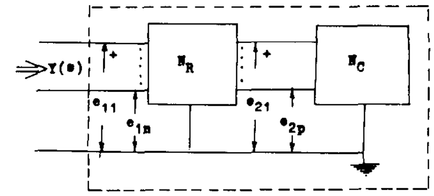

Fig. 1. Network block diagram.

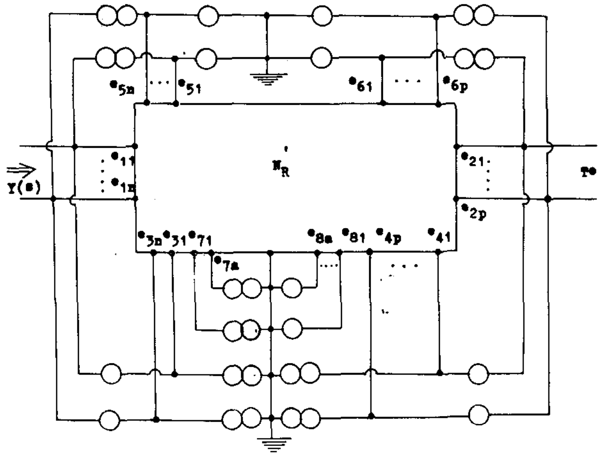

Fig. 2. Subnetwork N_R.

$$E = \begin{bmatrix} e_1' \\ e_2' \\ \cdot \\ \cdot \\ \cdot \\ e_8' \end{bmatrix}$$

where

$$e_j' = [e_{j1}', e_{j2}', \cdots, e_{jn}']^t, \qquad \text{for } j = 1, 3, \text{ and } 5$$

$$e_j' = [e_{j1}', e_{j2}', \cdots, e_{jp}']^t, \qquad \text{for } j = 2, 4, \text{ and } 6$$

and

$$e_j' = [e_{j1}', e_{j2}', \cdots, e_{ja}']^t, \qquad \text{for } j = 7 \text{ and } 8.$$

Similarly, the port current vector I of N_R' will be noted. Then the short-curcuit conductance matrix G_{NR}' of N_R' will relate I and E as

$$I = G_{NR}' \cdot E = (g_{jk}) E \tag{3}$$

where g_{jk} is the submatrix of G_{NR}' formed by the rows and columns corresponding to i_j' and e_k'. G_{NR}' will be constructed so as to be a real symmetric hyperdominant matrix.

Now the connection of nullator-norator pairs to N_R' imposes the following constraints

$$e_3' = e_1' = e_1$$

$$e_4' = e_2' = e_2$$

$$e_5' = e_6' = e_8' = 0$$

$$i_8' = 0$$

$$i_1 = i_1' + i_5'$$

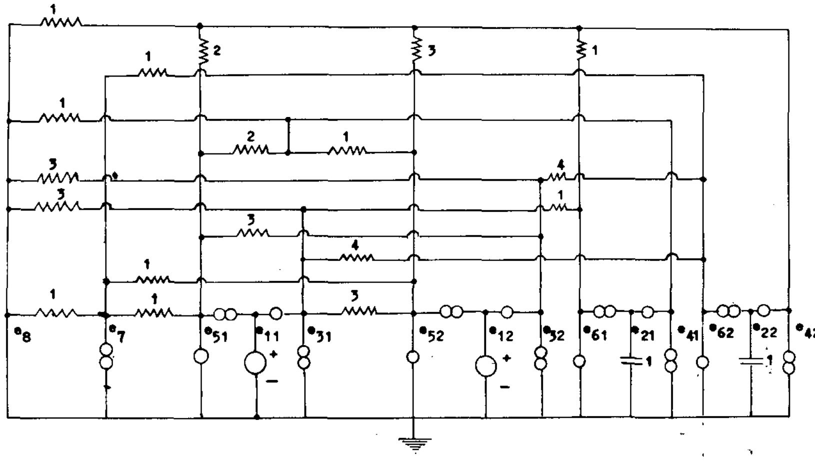

Fig. 3. Network realized in example. Active RC n-port synthesis using nullators and norators. All values mhos and farads.

and

$$i_2 = i_2' + i_6'. \tag{4}$$

Further, let

$$g_{ij} = [0], \quad \text{for } i = 1 \text{ to } 8, \quad j = 1 \text{ and } 2 \tag{5}$$

and

$$g_{87} = [-u].$$

Then from (1) to (5) we get

$$\begin{bmatrix} J & H \\ -CG & -CF \end{bmatrix} = \begin{bmatrix} g_{53} & g_{54} \\ g_{63} & g_{64} \end{bmatrix} + \begin{bmatrix} g_{57} \\ g_{67} \end{bmatrix} [g_{83} \;\; g_{84}]. \tag{6}$$

It may be noted that the first term on the left-hand side of (6) is a matrix of nonpositive elements with no restrictions on their values and the second term is a matrix of nonnegative elements, again with no restrictions on the element values. By choosing C and then splitting the matrix on the left-hand side of (6) into two matrices satisfying the above requirements, one can obtain the elements of G_{NR}' and hence N_R'. We next give a procedure to get a simple decomposition of G_{NR} as indicated by (6) and to obtain a bound on the maximum number of nullator-norator pairs required for the realization of any arbitrary matrix $Y(s)$

We first split the left-hand side of (6) as the sum of two matrices A and B where A contains exactly all the nonpositive elements and B contains all the nonnegative elements. Now by suitably adding elements to both A and B we can reduce B to a rank 1 matrix. The resulting matrices A' and B' will still retain the nonpositive and nonnegative character of A and B. We then identify A' and B' as

$$A' = \begin{bmatrix} g_{53} & g_{54} \\ g_{63} & g_{64} \end{bmatrix} \tag{7}$$

and

$$B' = \begin{bmatrix} g_{57} \\ g_{67} \end{bmatrix} [g_{83} \;\; g_{84}]. \tag{8}$$

Since B' is of rank 1, it is quite simple to obtain a decomposition such that $[g_{57} \;\; g_{67}]^t$ and $[g_{83} \;\; g_{84}]$ are vectors of nonpositive entries. The number a of nullator-norator pairs connected to ports 7 and 8 then becomes 1; thus the maximum number of nullator-norator pairs required to realize an arbitrary $Y(s)$ matrix by this method is $2n + 2p + 1$.

From (7) and (8) the entire G_{NR}' matrix can be constructed, taking care to see that it is a hyperdominant matrix.

Example

Consider the $Y(s)$ matrix given in [9, example (1)]

$$Y(s) = \frac{1}{s^2 + s + 1} \begin{bmatrix} 3s^2 + s + 2 & -(s+1) \\ -2(s-1) & 3s^2 + s + 3 \end{bmatrix}.$$

One set of matrices F, G, H, and J are

$$F = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \quad G = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} -1 & -1 \\ 0 & -2 \end{bmatrix} \quad J = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}.$$

Choosing

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad g_{87} = -1$$

we get

$$G_{NR} = A' + B'$$

$$= \begin{bmatrix} 0 & -3 & -2 & -2 \\ -3 & 0 & -1 & -3 \\ -1 & 0 & 0 & -1 \\ -4 & -4 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \end{bmatrix} [-3 \;\; -3 \;\; -1 \;\; -1] \tag{9}$$

$$= \begin{bmatrix} g_{53} & g_{54} \\ g_{63} & g_{64} \end{bmatrix} + \begin{bmatrix} g_{57} \\ g_{67} \end{bmatrix} [g_{83} \;\; g_{84}]. \tag{10}$$

Using (9) and (10) the entire G_{NR}' matrix is constructed taking care to see that it is hyperdominant. The final network obtained after embedding nullator-norator pairs on N_R' and connecting N_c is shown in Fig. 3. It should be noted that we have used 9 nullator-norator pairs and 18 resistors as compared to the 11 nullator-norator pairs and 17 resistors in [9, example (1)].

Conclusion

A simple and systematic method of synthesizing active *RC* networks with nullator-norators pairs is given. The method was developed using simple matrix manipulation. A bound on the maximum number of nullator-norator pairs for the realization of any arbitrary $Y(s)$ matrix was obtained.

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Signal Flow Graphs–Computer-Aided System Analysis and Sensitivity Calculations

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Abstract–Concepts which promise to extend many fundamental results of network theory to general systems are introduced. The basis for these extensions is the introduction of two matrices, the summing matrix S and the branching matrix B, which completely describe the topology of a signal flow graph. This leads to a formulation of system equations in terms of submatrices of the S- and B-matrices suitable for digital-computer programming. Consequently, many computer-aided circuit analysis and design programs can now be employed for the computer-aided analysis and design of systems representable by signal flow graphs. This formulation also leads to a straightforward algorithm for obtaining the system gain, an alternate to using Mason's gain formula. Furthermore, the power of this formulation, and its strong relation to network theory, is demonstrated by the derivation of a theorem similar to Tellegen's theorem in network theory. The theorem depends only on the topological properties of the summing and branching matrices and not on the functional relationships between the branch variables. An application of this theorem leads to a definition of adjoint signal flow graphs for both static and dynamic systems. From this, a sensitivity calculation for output variables with respect to a system parameter is developed which is equivalent to the analysis of two signal flow graphs, the original one and its adjoint.

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Introduction

Signal flow graphs were first introduced by Mason [1] and have been used extensively in representing the structure of relationships between variables of a system. Mason's general gain formula permits calculating the system gain directly from the signal flow graphs. However, the formula does not lead to an efficient computer program. Thus its use in a computer-aided system analysis program is somewhat tenuous. In this paper, two matrices, the summing matrix S and the branching matrix B, are introduced, which completely describe the topology of a signal flow graph. A