The Modified Circuit Matrix of an n-Port Network and Its Applications

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***Abstract*—Certain properties of the modified circuit matrix useful in the analysis and synthesis of n-port networks are outlined. A sufficient condition based on the modified circuit matrix is given for the proper pseudoseries connection of n-port networks. The usefulness of the concept of the modified circuit matrix in the synthesis of networks is demonstrated by giving a new approach to the realization of two-element-kind two-port networks.**

I. Introduction

THE MODIFIED cutset and the modified circuit matrices were first introduced by Cederbaum in connection with the generation of equivalent n-port networks [1]. The properties of the modified cutset matrix have subsequently been investigated in more detail, and they have been used to obtain a necessary and sufficient condition for the proper parallel connection of n-port networks [2]–[4], and also to obtain an improved method for the generation of a class of continuously equivalent networks of a given n-port network [4]. In this paper, the properties of the modified circuit matrix are investigated. Several of the results obtained in this paper are similar to those obtained in [4].

In Section II, certain properties of the modified circuit matrix useful from the point of view of synthesis of n-port networks are outlined. In Section III, a sufficient condition is obtained for the proper pseudoseries connection of n-port networks. A procedure is given in Section IV for the realization of (2×2) real symmetric dominant matrices by networks all having a prescribed modified circuit matrix. The results of Section IV are used in Section V to obtain a procedure for the realization of the impedance matrices of two-element-kind two-port networks.

II. Properties of the Modified Circuit Matrix

We consider an RLC n-port network with a current source connected to each one of its ports. Each current source, along with the impedance in series with it, is replaced by an edge in the linear graph of the network. Where such an impedance does not exist in the network, we consider an element of zero impedance to be present. Let the graph so formed be in one part and have e edges and v vertices. All the edges corresponding to current sources can be included in a cotree of the graph of an n-port network permitting connection to n independent current sources. We choose a tree of the graph satisfying this property and refer to the links representing the current sources as port chords. Let $B_f = \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix}$ be the fundamental circuit matrix of the network with respect to this tree, where the rows of the submatrix B_1 correspond to the port chords and those of the submatrix B_2 correspond to the remaining $(e - v + 1 - n)$ chords, which may be referred to as nonport chords. If Z_e is the diagonal matrix of edge impedances, $Z_{11} = B_1 Z_e B_1'$, $Z_{12} = B_1 Z_e B_2' = Z_{21}'$, and $Z_{22} = B_2 Z_e B_2'$, then the modified circuit matrix B and the open-circuit impedance matrix Z of the n-port network are given [1] as

$$B = B_1 - Z_{12} Z_{22}^{-1} B_2 \tag{1}$$

and

$$Z = Z_{11} - Z_{12} Z_{22}^{-1} Z_{21} = B Z_e B'. \tag{2}$$

For the singular cases where Z_{22}^{-1} does not exist, the modified circuit matrix also does not exist. Barring these cases, if I_e, I_p, I_n, V_e, and V_p, respectively, refer to the column matrices of edge currents, port currents (source currents), currents in the nonport chords, voltages across edge impedances and port voltages, respectively, then we have

$$I_e = B' I_p \tag{3}$$

$$V_p = B V_e \tag{4}$$

and

$$I_n = -Z_{22}^{-1} Z_{21} I_p. \tag{5}$$

Equations (2), (3), and (4) emphasize the similarity in the roles of the modified circuit matrix and the fundamental circuit matrix. Where the number of ports equals the number of chords, the modified circuit matrix is the same as the fundamental circuit matrix. Unlike B_f, B is not a purely topological matrix and it depends, in general, on both the topology and the edge impedances.

Theorem 1

The element b_{ij} of the modified circuit matrix B is equal to the current through the edge corresponding to column j when port i is excited with a source of unit current and all the other ports open-circuited.

Proof: The theorem follows from (3).

Theorem 2

Let $M = [m_{ij}]$ be a matrix of order $n \times (e - v + 1 - n)$ such that 1) the row and column ordering of M corresponds to the ordering of the rows of B_1 and B_2, respectively, and 2) m_{ij} is equal to the current through the jth

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nonport chord when port i is excited with a source of unit current and all the other ports are open-circuited. Then $M = -Z_{12}Z_{22}^{-1}$ and $B = B_1 + MB_2$.

Proof: Let M_i be the ith row of M. If port i is excited with a source of unit current and all the other ports open-circuited, then we have from hypothesis

$$I_n = M_i'. \tag{6}$$

But from (5),

$$I_n = -Z_{22}^{-1}Z_{21}I_p$$
$$= -Z_{22}^{-1}Z_{21}\begin{bmatrix}0\\0\\\vdots\\0\\1\\0\\\vdots\\0\end{bmatrix} \leftarrow i\text{th row} \tag{7}$$
$$= -(i\text{th column of } Z_{22}^{-1}Z_{21}).$$

From (6) and (7),

$$M_i = -(i\text{th row of } Z_{12}Z_{22}^{-1}),$$

similarly for the other rows of M. Hence, it follows that

$$M = -Z_{12}Z_{22}^{-1}, \tag{8}$$

and from (1),

$$B = B_1 + MB_2. \tag{9}$$

Theorem 3

If Z_e be the edge impedance matrix of a given n-port network and B its modified circuit matrix, then 1) $BZ_eB_2' = 0$, and 2) $BZ_eB_1' = Z$, where Z is the open-circuit impedance matrix of the network.

Proof:

1)
$$\begin{aligned} BZ_eB_2' &= (B_1 - Z_{12}Z_{22}^{-1}B_2)Z_eB_2' \\ &= Z_{12} - Z_{12}Z_{22}^{-1}Z_{22} \\ &= 0. \end{aligned} \tag{10}$$

2)
$$\begin{aligned} BZ_eB_1' &= (B_1 - Z_{12}Z_{22}^{-1}B_2)Z_eB_1' \\ &= Z_{11} - Z_{12}Z_{22}^{-1}Z_{21} \\ &= Z. \end{aligned} \tag{11}$$

Theorem 4

Given the modified circuit matrix of an n-port network having specified edge and port configurations and orientations, then the matrix M is uniquely specified.

Proof: The proof follows from Theorems 1 and 2.

Theorem 5

Let for a given n-port network N_1, B_2 be the nonport chord submatrix of the fundamental circuit matrix and B the modified circuit matrix. If any diagonal matrix Z_{e2} satisfies the equation $BZ_{e2}B_2' = 0$, then the modified circuit matrix of an n-port network N_2 with Z_{e2} as its edge impedance matrix and having identical port and edge configurations and orientations as N_1 is also equal to B.

Proof: From Theorem 4, it follows that the matrix M of network N_1 is uniquely determined. Its modified circuit matrix B is then equal to $(B_1 + MB_2)$ by Theorem 2, and by hypothesis we have

$$(B_1 + MB_2)Z_{e2}B_2' = 0,$$

or

$$(B_1Z_{e2}B_2')(B_2Z_{e2}B_2')^{-1} = -M. \tag{12}$$

Since N_2 has the same edge and port configurations and orientations as N_1, the topological matrices B_1 and B_2 are the same for the two networks. The matrix on the left-hand side of (12) can then be recognized to be the $(Z_{12}Z_{22}^{-1})$ matrix for N_2. Hence, its modified circuit matrix is equal to $(B_1 + MB_2)$ or B, i.e., the modified circuit matrix of N_1.

From the above theorem, it follows that the class of all n-port networks having the same modified circuit matrix as a given n-port network can be obtained from the solution of $BZ_eB_2' = 0$. This result will be used in Section IV.

III. Pseudoseries Connection of n-Port Networks

One approach to the solution of the problem of realization of a real symmetric matrix as the open-circuit resistance matrix of an n-port network is to realize the inverse of the given matrix as the short-circuit conductance matrix of an n-port network. Extension of this approach to the realization of the open-circuit impedance matrix Z of an RLC n-port network requires the realization of the parameter matrices or the residue matrices (if they are real) of the given matrix Z and a suitable connection of the appropriate n-port networks realizing the component matrices of Z, so that such a combination has an open-circuit impedance matrix equal to Z. Guillemin [5] observes that such an extension is not feasible since "if two or all three of these open-circuit resistance matrices are given and the problem is to find an RL or RLC network characterized by them, then the realizations found in this way are of no use, for there is no way in which the separate single-element-kind realizations can be connected to form the desired result." On the other hand, Slepian [6] emphasizes the need for finding a solution to this problem when he poses the question of defining an addition of two n-port networks N_1 and N_2 to obtain a new n-port network N_3 and determining the conditions on N_1 and N_2 so that the open-circuit impedance matrix of N_3 is equal to the sum of the open-circuit impedance matrices Z_1 and Z_2 of N_1 and N_2. In this section, a partial answer to this question is offered.

We consider two n-port networks N_1 and N_2 with identical port and edge configurations and orientations and having Z_1 and Z_2 for their open-circuit impedance matrices. From these we form a third n-port network N_3 also having the same edge and port configurations and orientations, but having the impedance of each edge as the sum of the impedances of the corresponding edges of N_1 and N_2. We shall refer to N_3 as the *pseudoseries combination* of N_1 and N_2. We now consider the conditions under which the open-circuit impedance matrix Z_3 of the network N_3 is equal to the sum of the open-circuit impedance matrices of N_1 and N_2. When N_3 has such a property, we qualify it as the *proper pseudoseries combination* of N_1 and N_2.

Let Z_{e1} be the diagonal edge impedance matrix of an n-port network N_1 having B as its modified circuit matrix and let Z_{e2} be the diagonal edge impedance matrix of another n-port network N_2 having the same topological configuration as N_1 and also the same modified circuit matrix B. It now follows that

$$\begin{aligned} B(Z_{e1} + Z_{e2})B_2' &= BZ_{e1}B_2' + BZ_{e2}B_2' \\ &= 0. \end{aligned} \tag{13}$$

The matrix $(Z_{e1} + Z_{e2})$ represents the edge impedance matrix of a network N_3, which is the pseudoseries combination of N_1 and N_2. From Theorem 5 it is seen that N_3 also has the same modified circuit matrix B. If Z_1, Z_2, and Z_3 are the open-circuit impedance matrices of N_1, N_2, and N_3, then we have

$$\begin{aligned} Z_3 &= BZ_{e3}B' \\ &= B(Z_{e1} + Z_{e2})B' = BZ_{e1}B' + BZ_{e2}B' = Z_1 + Z_2. \end{aligned} \tag{14}$$

Hence, N_3 is the proper pseudoseries combination of N_1 and N_2. Thus, it can be concluded that a sufficient condition for the proper pseudoseries combination of N_1 and N_2 is that their modified circuit matrices be equal. It can also be shown that this condition is necessary for the proper pseudoseries combination of N_1 and N_2 if N_1 and N_2 contain only positive R, L, and C elements [7]. This condition for the proper pseudoseries combination is similar to the one based on the modified cutset matrix obtained for the proper parallel combination of two n-port networks without internal vertices [3], [4]. It may, however, be noted that pseudoseries combination is not the dual of parallel combination, when internal vertices are present.

IV. Realization of a (2 × 2) Real Dominant Matrix

In this section, we consider the problem of realization of a (2 × 2) real dominant matrix as the open-circuit resistance matrix of a two-port network having a prescribed modified circuit matrix.

Consider the two-port resistive network shown in Fig. 1, where p, q, a, b, c, d refer to the resistances of the respective edges. The modified circuit matrix of this two-port network can then be obtained as

$$B = \begin{array}{c} \\ \end{array}\begin{bmatrix} \overset{e_{1'3}}{1} & \overset{e_{2'4}}{0} & \overset{e_{1'4}}{-m_1} & \overset{e_{34}}{m_1} & \overset{e_{1'2'}}{-(1-m_1)} & \overset{e_{32'}}{(1-m_1)} \\ 0 & 1 & -(1-m_2) & -m_2 & (1-m_2) & m_2 \end{bmatrix} \tag{15}$$

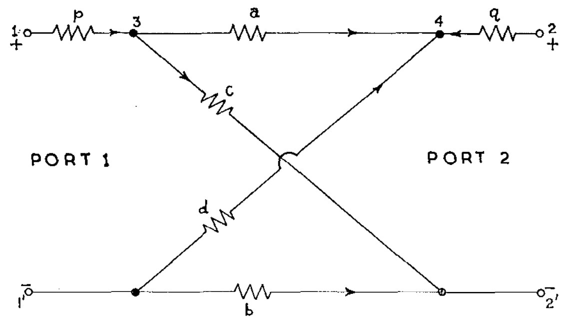

Fig. 1. Two-port resistive network.

where $m_1 = (b + c)/(a + b + c + d)$ and $m_2 = (b + d)/(a + b + c + d)$, and m_1 and m_2 are positive.

Given the modified circuit matrix as in (15) and a real symmetric open-circuit resistance matrix Z as below, let it be required to find a two-port network in the form shown in Fig. 1 having these specifications.

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}. \tag{16}$$

Let $B_f = \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix}$ be the fundamental circuit matrix of this network with respect to the tree comprising the edges e_{34}, $e_{1'2'}$, and $e_{32'}$ with B_1 and B_2 as defined in Section II.

Let

$$R = \begin{bmatrix} p & & & & & \\ & q & & 0 & & \\ & & d & & & \\ & & & a & & \\ & 0 & & & b & \\ & & & & & c \end{bmatrix},$$

the edge resistance matrix, satisfy the two equations

$$BRB_1' = Z \tag{17}$$

$$BRB_2' = 0. \tag{18}$$

This implies that the modified circuit matrix and the open-circuit resistance matrix of the two-port network having R as its edge resistance matrix are equal to B and Z, respectively. Equations (17) and (18) together can be put in the following form.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1-m_1 & 1-m_1 \\ 0 & 1 & 0 & m_2 & 0 & m_2 \\ 0 & 0 & 0 & -m_1 & 0 & 1-m_1 \\ 0 & 0 & 0 & 0 & -(1-m_2) & m_2 \\ 0 & 0 & -m_1 & -m_1 & 1-m_1 & 1-m_1 \\ 0 & 0 & -(1-m_2) & m_2 & -(1-m_2) & m_2 \end{bmatrix} \begin{bmatrix} p \\ q \\ d \\ a \\ b \\ c \end{bmatrix} = \begin{bmatrix} z_{11} \\ z_{22} \\ z_{12} \\ z_{21} \\ 0 \\ 0 \end{bmatrix}. \tag{19}$$

Given the open-circuit resistance matrix Z and the modified circuit matrix B, a unique solution of the six resistance values does not exist, as the coefficient matrix in the above equations has a rank equal to 5. We can, however, choose one of the resistances, say c, as an arbitrary parameter and express the other resistances p, q, a, b, and d in terms of c, m_1, m_2, z_{11}, z_{12}, and z_{22} as follows.

$$\begin{aligned} a &= \frac{(1-m_1)c}{m_1} - \frac{z_{12}}{m_1} \\ b &= \frac{m_2 c}{(1-m_2)} - \frac{z_{12}}{(1-m_2)} \\ d &= \frac{(1-m_1)m_2 c + z_{12}(m_1 - m_2)}{m_1(1-m_2)} \\ p &= z_{11} + \frac{(1-m_1)z_{12}}{(1-m_2)} - \frac{(1-m_1)c}{(1-m_2)} \\ q &= z_{22} + \frac{m_2 z_{12}}{m_1} - \frac{m_2 c}{m_1}. \end{aligned} \tag{20}$$

For a proper realization, it is required that p, q, a, b, and d be non-negative for a non-negative c. This restriction leads to the following inequalities:

$$\begin{aligned} c &\geq \frac{z_{12}}{(1-m_1)} \\ c &\geq \frac{z_{12}}{m_2} \\ c &\geq \frac{z_{12}(m_2 - m_1)}{(1-m_1)m_2} \\ c &\leq \frac{z_{11}(1-m_2)}{(1-m_1)} + z_{12} \\ c &\leq \frac{z_{22}m_1}{m_2} + z_{12}. \end{aligned} \tag{21}$$

If m_1 and m_2 or, in other words, B is specified, then choosing a non-negative c satisfying the above inequalities and using (20), a proper two-port realization of Z having the specified modified circuit matrix and the configuration shown in Fig. 1 can be obtained.

If it is not possible to obtain a non-negative c satisfying the inequalities in (21), it only means that the given dominant matrix Z cannot be realized with the specified modified circuit matrix. However, it can be realized with some other modified circuit matrix.

We now consider a different aspect of the problem. Given a Z matrix, we wish to determine an appropriate modified circuit matrix, which leads to a proper realization of a given Z in the form of the network shown in Fig. 1. One solution of this problem, which is presented below, is adequate for our purpose.

Case A: $z_{22} \geq z_{11}$

Let m_1 and m_2 satisfy the following relations:

$$m_2 \geq m_1 \geq \tfrac{1}{2};\ z_{11}(1-m_2) \geq m_1 |z_{12}|. \tag{22}$$

Then the following relations hold.

1) $z_{12} < 0$:

$$\frac{z_{12}}{1-m_1} \leq \frac{z_{12}}{m_2} \leq \frac{z_{12}(m_2 - m_1)}{m_2(1-m_1)} \leq 0 \leq \frac{z_{11}(1-m_2)}{(1-m_1)} + z_{12} \leq \frac{z_{22}m_1}{m_2} + z_{12}$$

or

2) $z_{12} > 0$:

$$0 \leq \frac{z_{12}(m_2 - m_1)}{m_2(1-m_1)} \leq \frac{z_{12}}{m_2} \leq \frac{z_{12}}{1-m_1} \leq \frac{z_{11}(1-m_2)}{(1-m_1)} + z_{12} \leq \frac{z_{22}m_1}{m_2} + z_{12}.$$

A non-negative c such that

$$\frac{z_{12}}{1-m_1} \leq c \leq \frac{z_{11}(1-m_2)}{1-m_1} + z_{12} \tag{23}$$

satisfies the constraints (21).

Case B: $z_{11} \geq z_{22}$

Let m_1 and m_2 satisfy the following relations:

$$\tfrac{1}{2} \geq m_2 \geq m_1;\ z_{22}m_1 \geq (1-m_2)\,|z_{12}|. \tag{24}$$

Then the following relations hold.

1) $z_{12} < 0$:

$$\frac{z_{12}}{m_2} \leq \frac{z_{12}}{1-m_1} \leq \frac{z_{12}(m_2 - m_1)}{(1-m_1)m_2} \leq 0 \leq \frac{z_{22}m_1}{m_2} + z_{12} \leq \frac{z_{11}(1-m_2)}{(1-m_1)} + z_{12}$$

or

2) $z_{12} > 0$:

$$0 \leq \frac{z_{12}(m_2 - m_1)}{(1-m_1)m_2} \leq \frac{z_{12}}{1-m_1} \leq \frac{z_{12}}{m_2} \leq \frac{z_{22}m_1}{m_2} + z_{12} \leq \frac{z_{11}(1-m_2)}{(1-m_1)} + z_{12}.$$

A non-negative c such that

$$\frac{z_{12}}{m_2} \leq c \leq \frac{z_{22}m_1}{m_2} + z_{12} \tag{25}$$

satisfies the constraints (21).

Thus, depending on the relative magnitudes of z_{11} and z_{22} in a given Z, m_1 and m_2 may be chosen to satisfy either (22) or (24) to ensure a proper realization of the matrix Z. Since $z_{22}/|z_{12}| \geq 1$ and $z_{11}/|z_{12}| \geq 1$, this is always possible to do. It must be noted that there exists, in general, a large number of sets of m_1 and m_2 satisfying (22) or (24) and, hence, a large number of realizations for a given Z is possible. If the matrix is just dominant, i.e., z_{11} or $z_{22} = |z_{12}|$, then the only possible choice is $m_2 = m_1 = \frac{1}{2}$. It may be further noted that for every appropriate set of values for m_1 and m_2 a large number of equivalent realizations can be obtained by choosing any value of c over a continuous range specified by (23) or (25).

The ideas presented here will be used in the next section to obtain a procedure for the synthesis of two-element-kind two-port networks.

V. Synthesis of Two-Element-Kind Two-Port Networks

In light of the result obtained in Section III on the sufficient condition for the pseudoseries connection of a set of n-port networks, realization of the Z matrix of an RLC n-port network leads to the problem of realization of the residue matrices of Z by resistive n-port networks, all of them having the same modified circuit matrix. When the residue matrices are not real, the parameter matrices of Z should be obtained, but determining the parameter matrices is not always easy. In this section, we consider the synthesis of two-element-kind two-port networks, since for two-element-kind networks real residue matrices can be conveniently set up and a synthesis procedure for two-port networks has been established in Section IV.

We first show that a set of (2×2) real symmetric dominant open-circuit resistance matrices $Z_1, Z_2, \cdots, Z_p$ can always be realized by resistive networks having the form shown in Fig. 1 and having the same modified circuit matrix. In the discussion that follows, a quantity pertinent to Z_k is distinguished by the superscript (k). First, we assume that for each matrix Z_k,

$$z_{11}^{(k)} \geq z_{22}^{(k)}. \tag{26}$$

Let ϵ be the minimum of

$$\left\{\frac{z_{22}^{(k)}}{|z_{12}^{(k)}|}\right\} \quad \text{for all} \quad k = 1, 2, \cdots, p.$$

It is then clear that m_1 and m_2, chosen to satisfy the following inequalities, meet the requirements of (29) for each Z_k and, hence, form an appropriate set for the proper realization of $Z_1, \cdots, Z_p$.

$$\epsilon \geq \frac{(1 - m_2)}{m_1} \tag{27}$$
$$\tfrac{1}{2} \geq m_2 \geq m_1.$$

If the requirement of (26) is not satisfied by any matrix Z_k, then we can always split Z_k as $Z_k' + Z_k''$, where Z_k'' is a diagonal matrix having the form

$$\begin{bmatrix} r_{11} & 0 \\ 0 & r_{22} \end{bmatrix},$$

and Z_k' a suitable dominant matrix satisfying the requirement of (26). We then consider Z_k' along with other matrices of the given set for purposes of determing appropriate m_1 and m_2. After a network realizing Z_k' is found, the resistances in series with ports 1 and 2 are increased by r_{11} and r_{22} to obtain a realization corresponding to Z_k. It is obvious that the new network realizing Z_k has the same modified circuit matrix as the one realizing Z_k'.

In general, a continuous set of values over a certain range can be found for m_1 and m_2 to satisfy the inequalities of (27). As pointed out in the last section, when one of the matrices of the given set is just dominant, then $\epsilon = 1$ and the choice is limited to $m_1 = m_2 = \frac{1}{2}$. For a chosen set of values for m_1 and m_2, once again a large number of continuously equivalent networks can be obtained by choosing different values of c in the interval given by (25).

Next, we consider the question of realization of two-element-kind two-port networks. Each entry z_{ij} in the open-circuit impedance matrix Z of a two-element-kind two-port network can be expressed as follows in a general case.

LC Network

$$z_{ij} = z_{ij}^{(1)} + \frac{z_{ij}^{(2)}}{s} + \sum_{r=3}^{p} \frac{2z_{ij}^{(r)}s}{s^2 + \omega_r^2}.$$

RL Network

$$z_{ij} = sz_{ij}^{(1)} + z_{ij}^{(2)} + \sum_{r=3}^{p} \frac{sz_{ij}^{(r)}}{s + \sigma_r}.$$

RC Network

$$z_{ij} = z_{ij}^{(1)} + \frac{z_{ij}^{(2)}}{s} + \sum_{r=3}^{p} \frac{z_{ij}^{(r)}}{s + \sigma_r}.$$

The (2×2) real matrices $[z_{ij}^{(1)}], [z_{ij}^{(2)}], \cdots, [z_{ij}^{(p)}]$ are termed the residue matrices. When the residue matrices are dominant, they can be realized by two-port networks, all of them having the same configuration shown in Fig. 1 and having the same modified circuit matrix by the procedure outlined above. All the resistances of the network realizing a particular residue matrix are then multiplied by the appropriate function of s and the resistive network converted into a two-element-kind network. The pseudoseries combination of all these networks will then have an open-circuit impedance matrix equal to the given matrix Z.

If the residue matrices are not dominant, then the impedance level of the transfer impedance may be scaled down by an appropriate factor so that the residue matrices are dominant. A two-port network having this modified Z matrix retains the same poles and zeros for the driving-point and transfer impedances as specified. However, the transfer impedance is realized with the appropriate scale factor.

Example 1

We next consider the realization of the following open-circuit impedance matrix Z.

$$Z = \begin{bmatrix} \dfrac{3s^2 + 12s + 4}{s^2 + s} & \dfrac{-s^2 - 2}{s^2 + s} \\[2ex] \dfrac{-s^2 - 2}{s^2 + s} & \dfrac{4s^2 + 12s + 3}{s^2 + s} \end{bmatrix}.$$

This can be split up as follows.

$$Z = \frac{1}{s}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} + \frac{1}{s+1}\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$
$$= \frac{1}{s} Z_1 + Z_2 + \frac{1}{s+1} Z_3.$$

To make $z_{11} \geq z_{22}$ for all the matrices, we split Z_2:

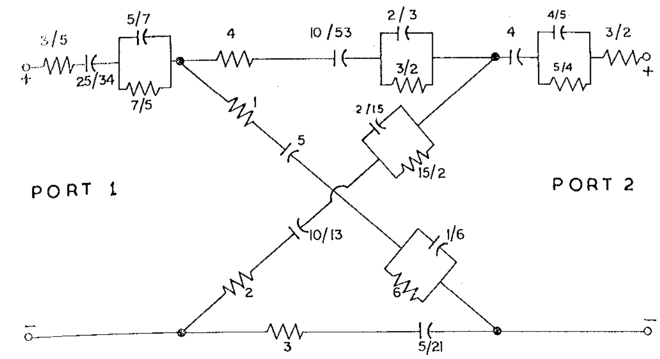

Fig. 2. Network realizing the Z matrix of Example 1.

$$Z_2 = Z_2' + Z_2'' = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

We next consider the realization of Z_1, Z_2', and Z_3 by two-port networks having the form shown in Fig. 1 and the same modified circuit matrix.

$$\epsilon = \tfrac{3}{2}.$$

Choose $m_1 = 0.4$ and $m_2 = 0.5$.

The following resistance values for the resistive two-port networks realizing Z_1, Z_2', and Z_3 are then obtained.

Network I: Realizing Z_1

$$a = \tfrac{53}{10};\ b = \tfrac{21}{5};\ c = \tfrac{1}{5};\ d = \tfrac{13}{10};\ p = \tfrac{34}{25};\ q = \tfrac{1}{4}.$$

Network II: Realizing Z_2'

$$a = 4;\ b = 3;\ c = 1;\ d = 2;\ p = \tfrac{3}{5};\ q = \tfrac{1}{2}.$$

Network III: Realizing Z_3

$$a = \tfrac{3}{2};\ b = 0;\ c = 6;\ d = \tfrac{15}{2};\ p = \tfrac{7}{5};\ q = \tfrac{5}{4}.$$

The network realizing $Z_2 = Z_2' + Z_2''$ can be obtained from Network II by connecting an additional resistance of 1 ohm in series with port 2 of Network II. The networks realizing $(1/s)Z_1$, Z_2, and $Z_3/(s+1)$ can be obtained by converting the resistances of the networks realizing Z_1, Z_2, and Z_3 into the appropriate impedance functions. The overall two-port network realizing the open-circuit impedance matrix Z is shown in Fig. 2.

VI. Conclusions

Certain properties of the modified circuit matrix useful in the analysis of n-port networks are obtained. Based on these properties, a sufficient condition is developed for the proper pseudoseries combination of n-port networks. The procedure given to realize the open-circuit impedance matrix of two-element-kind two-port networks demonstrates the usefulness of this criterion in network synthesis. Even though an unsymmetrical lattice realizing the Z matrix of Example 1 can possibly be obtained by other methods, the present approach has the feature that for a given Z matrix a large number of continuously equivalent networks can be obtained. The application of this method to the synthesis of higher order n-port networks and to the generation of equivalent n-port networks is under investigation.

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