Multiprocessor Fault Diagnosis Under Local Constraints

A. Das, K. Thulasiraman, V. K. Agarwal, and K. B. Lakshmanan

***Abstract*—In this correspondence, we study fault diagnosis of multiprocessor systems when fault constraints in the local domain of each processor are specified. We use the comparison-based model. A multiprocessor system S is *t-in -L diagnosable,* if given a syndrome, all faulty processors can be uniquely identified provided there are at most t faulty processors in the local domain $L(u_i) \cup \{u_i\}$ of every processor u_i in S, where $L(u_i)$ denotes the set of processors adjacent to u_i. First, we present certain basic results that lead to sffucient conditions for unique diagnosis of a system when certain fault constraints are satisfied in the local domain of each processor in the system. We then examine the t-in-L diagnosability of certain regular interconnected systems under the assumption that less than half of the total number of processors in the system are faulty. We also present diagnosis algorithms for these systems.**

***Index Terms*—Algorithms, distributed algorithm, fault diagnosis, graph theory, multiprocessor system.**

I. Introduction

Continuing advances in semiconductor technology have now made available large multiprocessor systems such as the hypercube systems. The increasing complexity of these systems poses challenging problems in ensuring their reliability. The problems of fault detection, diagnosis, and reconfiguration of multiprocessor systems have thus become active areas of intensive research in recent years. Various models of fault diagnosis have been studied, and significant algorithms and related complexity results have been reported [1]–[11].

In multiprocessor systems such as those implementable in very large scale integration (VLSI) and wafer-scale integration (WSI), the number of units in a system can be very large. Moreover, the commonly used system interconnection networks such as the rectangular grids are very symmetrical and sparse. When such a system is analyzed using the classical theory, the number of faulty processors permitted is very small in comparison to the number of units in the system. This shortcoming motivated the recent works on probabilistic diagnosis algorithms for sparsely interconnected systems [10], [11]. Our work in this paper is also motivated by the inadequacy of the classical approach when applied to large sparsely interconnected systems and the need for distributed diagnosis algorithms. We use the comparison-based model introduced by Chwa and Hakimi [5] in which all processors are assigned to perform the same task and the outputs of neighboring processors are then compared. Instead of a single global constraint, in this paper, we consider local constraints on the number of faulty processors in the neighborhood of each processor in the multiprocessor system.

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II. Preliminaries

A multiprocessor system consists of n independent processors $U = \{u_1, u_2, \cdots, u_n\}$. As we stated earlier, in the comparison model of multiprocessor fault diagnosis [5], all processors in S are assigned to perform the same task. Upon completion, the outputs of neighboring pairs of these processors are compared. The comparison assignment can be represented by an undirected graph $G = (U, E)$ where an edge e_{ij} belongs to E if and only if the outputs of u_i and u_j are compared. An outcome a_{ij} is associated with each pair of processors whose outputs are compared, where $a_{ij} = 0(1)$ if the outputs compared agree(disagree). Only permanent faults are considered, and as in [5] we assume that the outputs of a fault-free and a faulty processor always disagree, so that $a_{ij} = 0$ whenever both u_i and u_j are fault-free, and $a_{ij} = 1$ if one of u_i and u_j is fault-free and the other faulty. If both u_i and u_j are faulty then a_{ij} is unreliable. $L(u_i)$ denotes the set of neighbors of u_i, that is, the set of all processors adjacent to u_i. An edge that has a 0(1) outcome associated with it is referred to as a 0-link(1-link). Paths starting from processor u_i are said to be *distinct* if and only if they have no vertex in common other than u_i. The distance between two processors u_i and u_j is denoted by $d(u_i, u_j)$. A fault set $F \subseteq U$ is a *permissible fault* set for a set of fault constraints if F satisfies the requirements of the fault constraints. Given a syndrome, F is an *allowable fault set* if and only if F is a permissible fault set, and the assumption that the processors in F are faulty and the processors in $U - F$ are fault-free is consistent with the given syndrome. Given a fault set $F, S(F)$ denotes the set of syndromes that can be generated by F, and F^c denotes the set $U - F$. Given two fault sets F_1 and $F_2, F_1 \oplus F_2$ denotes the *symmetric difference* between F_1 and F_2. A system S is defined to be *t- in-L diagnosable* if given a syndrome, all faulty processors can be uniquely identified provided that there are at most t faulty processors in the local domain $L(u) \cup \{u\}$ for every processor u in S.

III. *t-in-L* Diagnosability

In this section, we study t-in-L diagnosability. Given a system S, we wish to determine the maximum value of t such that S is t-in-L diagnosable. Clearly, if we allow at most $\lfloor |L(u_i)|/2 \rfloor$ faults in $L(u_i) \cup \{u_i\}$ for each u_i, then a majority vote of the outcomes for each processor will correctly diagnose the faulty or fault-free status of each u_i. Interestingly, as we shall see in the following section, unique diagnosability of most regular systems of interest to us requires that we permit no more than $\lfloor |L(u_i)|/2 \rfloor + 1$ faults in $L(u_i) \cup \{u_i\}$ for each u_i. Thus, we confine our investigations to this case. We say that a system satisfies *local fault constraints* if for each u_i there are at most $\lfloor |L(u_i)|/2 \rfloor + 1$ faults in the local domain $L(u_i) \cup \{u_i\}$.

In this section we present certain basic properties of allowable fault sets corresponding to a given syndrome (Lemmas 1–3). These properties lead to sufficient conditions (Theorem 1) for unique diagnosis of a system S that satifies the local fault constraints.

Lemma 1: Given a system S and a syndrome, let F_1 and F_2 be two distinct allowable fault sets for the given syndrome such that $F_1 \cup F_2 \neq U$, and for all processors $u \in U, L(u) - F_1$ and $L(u) - F_2$ are both nonempty. Then there exist processors $x, y \in U$ such that

1) $x \in U - (F_1 \cup F_2)$
2) $y \in F_1 \oplus F_2$
3) $2 \leq d(x, y) \leq 3$ and $d(x, y)$ is minimum among all x and y satisfying conditions 1 and 2.

Proof: Since $F_1 \cup F_2 \neq U$ the set $U - (F_1 \cup F_2)$ is nonempty. Furthermore, since F_1 and F_2 are distinct, there exists at least one processor that belongs to one fault set and is not contained in the other. Thus there exist processors in U satisfying conditions 1 and 2. Now let x and y be processors in U satisfying conditions 1 and 2 such that the distance $d(x, y)$ is minimum.

Assume $d(x, y) \geq 4$. Consider a processor w that is at a distance at most $\lceil d(x, y)/2 \rceil$ from both x and y. Since $L(w) - F_1$ and $L(w) - F_2$ are both nonempty, there exists a processor $z \in L(w)$ such that $z \in U - (F_1 \cup F_2)$ or $z \in F_1 \oplus F_2$. If $z \in U - (F_1 \cup F_2)$ then $d(z, y) \leq d(w, y) + 1 < d(x, y)$; if $z \in F_1 \oplus F_2$ then $d(x, z) \leq d(x, w) + 1 < d(x, y)$. In either case, the minimality of $d(x, y)$ is contradicted. Hence $d(x, y) \leq 3$.

To prove that $d(x, y) \geq 2$, we show that the assumption $d(x, y) = 1$ leads to a contradiction. Assume $d(x, y) = 1$. Then the link between x and y is a 0-link with respect to one fault set and a 1-link with respect to the other, contradicting the assumption that F_1 and F_2 share a common syndrome. □

Lemma 2: Let S be a system with test interconnection graph $G = (U, E)$ and which satisfies local fault constraints. Given a syndrome s_1 and two allowable fault sets F_1 and F_2 with $s_1 \in S(F_1) \cap S(F_2)$, the following conditions hold for every $x \in F_1 \oplus F_2$, where $|L(x)| = k$:

1) $|F_1 \cap F_2 \cap L(x)| \leq 1$.
2) $|(F_1 \oplus F_2) \cap L(x)| \geq k - 1$.

Proof: Without loss of generality, assume $x \in F_1 - F_2$. Then x is faulty with respect to F_1 and fault-free with respect to F_2. Let X denote the subset of processors in $L(x)$ that are fault-free with respect to F_2. The processors in X are all faulty with respect to F_1 since they have 0-links with x, and x is faulty with respect to F_1. $|X| \leq \lfloor k/2 \rfloor$ since there are at most $\lfloor k/2 \rfloor + 1$ faulty processors in $L(x) \cup \{x\}$. Furthermore, $|X| \geq \lceil k/2 \rceil - 1$, since the processors in $L(x) - X$ are all faulty with respect to F_2 and there are at most $\lfloor k/2 \rfloor + 1$ faulty processors in $L(x) \cup \{x\}$. Thus,

$$\lceil k/2 \rceil - 1 \leq |X| \leq \lfloor k/2 \rfloor.$$

Now consider the processors in $L(x) - X$. They are all faulty with respect to F_2. Now, if more than one processor in $L(x) - X$ is also faulty with respect to F_1, then the number of faulty processors in $L(x) \cup \{x\}$ with respect to F_1 is greater than $\lfloor k/2 \rfloor + 1$, contradicting our assumption that F_1 is a permissible fault set. This shows that condition 1 is true.

Since all processors in X are contained in $F_1 - F_2$ and all processors except at most one in $L(x) - X$ belong to $F_2 - F_1$, there are at least $|L(x)| - 1$ processors in $L(x)$ which also belong to $F_1 \oplus F_2$. Since $|L(x)| = k$, it follows that (2) holds. □

Lemma 3: Consider a system S with test interconnection graph $G = (U, E)$ in which the number of faulty processors is less than $|U|/2$. Let S satisfy the local fault constraints and $|L(u_i)| \geq 3$ for every $u_i \in U$. If s_1 is a syndrome, and F_1 and F_2 are two allowable fault sets with $s_1 \in S(F_1) \cap S(F_2)$, then we have the following:

i) There exist two processors $x, y \in U$ such that
   a) $x \in U - (F_1 \cup F_2)$
   b) $y \in F_1 \oplus F_2$
   c) $2 \leq d(x, y) \leq 3$ and $d(x, y)$ is minimum among all x and y satisfying conditions a) and b).
   d) If w lies on any shortest path between x and y and $d(w, y) = 2$ then there is exactly one path of length 2 between w and y.

ii) For every pair of processors x and y satisfying conditions a)–c) of i), condition id) holds for every shortest path between x and y.

Proof: Consider any processor $u_i \in U$. Since there are at most $\lfloor L(u_i)/2 \rfloor + 1$ faulty processors in the local domain $L(u_i) \cup \{u_i\}$, it follows that for any allowable fault set F, we have $|L(u_i) - F| \geq |L(u_i)| - \lfloor L(u_i)/2 \rfloor - 1$. So, $|L(u_i) - F| \geq \lfloor L(u_i)/2 \rfloor - 1$, if $|L(u_i)|$ is even; otherwise, $|L(u_i) - F| \geq \lfloor |L(u_i)/2| \rfloor$. But, by assumption, $|L(u_i)| \geq 3$. So $|L(u_i) - F| \geq 1$, for any allowable fault set F. Since both F_1 and F_2 are allowable fault sets, $L(u_i) - F_1$ and $L(u_i) - F_2$ are nonempty, and so by Lemma 1 there exist x and y satisfying conditions a)–c) of i. Thus, to show that i and ii hold, we need now only prove that if x and y are arbitrary processors in S satisfying conditions a)–c) of i, then condition d) is true.

Let x and y be any pair of processors in S satisfying conditions a)–c) of i. Now assume condition d) is not true. Then there exists a processor w lying on a shortest path between x and y with $d(w, y) = 2$ and there are two or more paths of length 2 between w and y. We note that w could be the processor x itself. We also observe that the system S, the fault sets F_1 and F_2, and the processor y satisfy the conditions of Lemma 2. Hence there exists at most one processor in $L(y)$ that belongs to $F_1 \cap F_2$ and all other processors belong to $F_1 \oplus F_2$. Since there are two or more paths of length 2 between w and y, there is at least one processor in $L(y)$ belonging to $F_1 \oplus F_2$ that is closer to x than y. If $w = x$, this will contradict condition ic); if $x \neq w$, this will contradict the minimality of $d(x, y)$. □

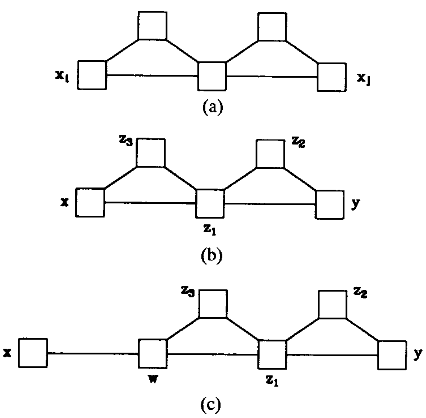

Fig. 1. Illustration for Theorem 1.

The following theorem provides sufficient conditions for t-in- L diagnosability.

Theorem 1: Consider a system S with test interconnection graph $G = (U, E)$ in which the number of faulty processors is less than $|U|/2$. Let S satisfy the local fault constraints and let $|L(u_i)| \geq 3$ for all $u_i \in U$. Then S is uniquely diagnosable if for any two processors x_i and x_j at distance 2 from each other, at least one of the following holds:

a) There are at least two vertex disjoint paths of length 2 between x_i and x_j.
b) The graph shown in Fig. 1(a) containing x_i and x_j is a subgraph of G.

Proof: We show that if S is not uniquely diagnosable then for some processor pair at distance 2 from each other neither a nor b is true. So assume that the system is not uniquely diagnosable. Then there exist two allowable fault sets F_1 and F_2 that share a common syndrome s. Thus there exist two processors x and y satisfying conditions a)–d) of i in Lemma 3.

Case 1: $d(x,y) = 2$. Clearly condition d) in i is violated if a holds for processors x and y. Now assume b holds and a does not hold for processors x and y. Consider Fig. 1(b). We observe that $z_1 \in F_1 \cap F_2$. By Lemma 2, $z_2 \in F_1 \oplus F_2$ which, in turn, implies that $z_3 \in F_1 \oplus F_2$. This again contradicts condition ic) of Lemma 3.

Case 2: $d(x,y) = 3$. Consider processor w, which lies on the shortest path between x and y, such that $d(w,y) = 2$. Clearly condition id) of Lemma 3 is violated if a holds for w and y. Now assume b holds and a does not hold for processors w and y. Consider Fig. 1(c). We observe that $z_1 \in F_1 \cap F_2$; otherwise, condition ic) of Lemma 3 is violated. Again by Lemma 2, $z_2 \in F_1 \oplus F_2$ which, in turn, implies that $z_3 \in F_1 \oplus F_2$. But this contradicts the minimality of $d(x,y)$.

Thus, if S is not uniquely diagnosable then there exist two processors at distance 2 from each other such that neither a nor b holds. Hence the theorem follows. □

The following is a straightforward consequence of Theorem 1.

Corollary 1.1: Let S be a system with interconnection graph $G = (U, E)$ in which the number of faulty processors is less than $|U|/2$ and $|L(u_i)| \geq 3$ for all $u_i \in U$. S is t-in-L diagnosable for $t = \lfloor \delta/2 \rfloor + 1$, where δ is the minimum degree of G, if for any two processors x_i and x_j at distance 2 from each other in G at least one of the following holds.

1) There are at least two vertex disjoint paths of length 2.
2) The graph shown in Fig. 1(a) containing x_i and x_j is a subgraph of G. □

IV. t-in-L Diagnosability of Regular Interconnected Systems

In this section we study the t-in-L diagnosability of certain regular interconnected systems—the closed rectangular, hexagonal, and octagonal grid systems and the hypercube systems. First, we consider the hypercube systems.

Theorem 2: Let S be a hypercube system containing 2^k processors, $k \geq 3$. The system S is t-in-L diagnosable for $t = \lfloor k/2 \rfloor + 1$ provided less than half the total number of processors in S are faulty.

Proof: The preceding result follows immediately from Corollary 1.1 and the observation that between any two processors at distance 2 from each other in a hypercube system there are two vertex disjoint paths of length 2. □

We now proceed to determine the maximum value of t for which other regular systems are t-in-L diagnosable under the assumption that less than half the total number of processors in these systems are faulty. Interestingly, we will see that the value of t is equal to $\lfloor \delta/2 \rfloor + 1$ in these cases too, where δ is the minimum degree.

Theorem 3: The maximum value of t that permits a closed rectangular grid S to be t-in-L diagnosable, given that less than half the processors in S are faulty, is 3.

Proof: The theorem is proved by contradiction. Assume there exist two permissible fault sets F_1 and F_2 sharing a common syndrome s, such that are at most three faulty processors in $L(u) \cup \{u\}$ for every processor u in S and F_1 and F_2 each contain less than half the total number of processors in the system. Since $|L(u)| = 4$ for every processor u, the system S and the two fault sets F_1 and F_2 satisfy the requirements of Lemma 3. Thus there exist processors x and y satisfying the conditions ia)—id) of this lemma. Assuming conditions ia), ib), and id) are satisfied by x and y, we arrive at a contradiction by showing that condition ic) is violated.

We observe that the status of all processors in $L(x)$ remains unchanged with respect to both F_1 and F_2 since x is fault-free in the presence of either fault set. This means that all processors that share a 1-link with x belong to $F_1 \cap F_2$. We also note that there cannot be a path of fault-free processors between x and y with respect to either fault set; otherwise, F_1 and F_2 cannot share a common syndrome.

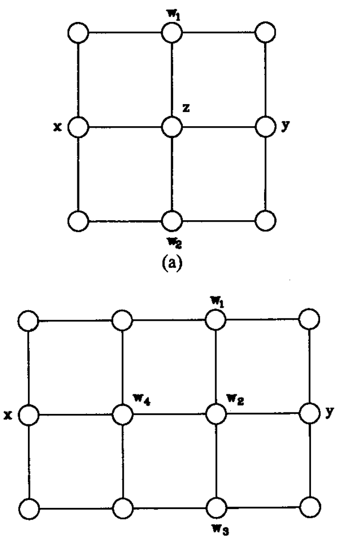

Fig. 2. Illustration for Theorem 3.

Case 1: $d(x,y) = 2$. Consider Fig. 2(a). All other cases with $d(x,y) = 2$ satisfying conditions ia), ib), and id) of Lemma 3 are symmetric to this case. The processor z must be faulty with respect to both fault sets; otherwise, there is a path of fault-free processors between x and y. Now the following subcases arise.

Case 1.1: w_1 or w_2 belongs to $F_1 \oplus F_2$. In this case, from Lemma 2, x is adjacent to a processor belonging to $F_1 \oplus F_2$; this contradicts the observation that processors in $L(x)$ belong to $F_1 \cap F_2$ or $(F_1 \cup F_2)^c$.

Case 1.2: w_1 and w_2 belong to $F_1 \cap F_2$. If both w_1 and w_2 are faulty with respect to F_1 and F_2, then since y is faulty in the presence of one of these fault sets, $L(z) \cup \{z\}$ contains more than three faulty processors with respect to F_1 or F_2; a contradiction.

Note that if w_1 or w_2 is in $(F_1 \cup F_2)^c$, then there would be two paths of length 2 between $y \in F_1 \oplus F_2$ and w_1 (or w_2) $\in (F_1 \cup F_2)^c$, contradicting condition id) of Lemma 3.

Case 2: $d(x,y) = 3$. We consider Fig. 2(b). All other cases with $d(x,y) = 3$, satisfying conditions ia), ib), and id) are symmetric to this case.

The processors w_2 and w_4 belong to $F_1 \cap F_2$; otherwise, the minimality of the distance $d(x,y)$ is violated. Similarly, w_1 and w_3 cannot be fault-free with respect to both F_1 and F_2 and so they belong to $F_1 \cup F_2$. Since w_1 and w_3 are at distance 3 from x and both have two disjoint paths of length 2 to w_4, by Lemma 3, they cannot belong to $F_1 \oplus F_2$. Thus w_1 and w_3 are in $F_1 \cap F_2$. But then $L(w_2) \cup \{w_2\}$ will have more than three faulty processors with respect to either F_1 or F_2.

From the preceding it follows that the system S is 3-in-L diagnosable given that less than half the processors in S are faulty. In addition, for a closed rectangular grid we can construct a syndrome and two allowable fault sets F_1 and F_2 such that for each $F_i, i = 1,2$, there exists a processor u with four faulty processors in $L(u) \cup \{u\}$. Thus the maximum value of t for which a closed rectangular grid is t-in-L diagnosable is 3. □

Theorem 4: Let S be a closed hexagonal grid or a closed octagonal grid system. Then the maximum value of t that permits S to be t-in-L diagnosable given that less than half the total number of processors in the system are faulty is k, where $k = 4$ and 5, respectively.

Proof: The conditions of Theorem 1 are satisfied for every pair of processors at distance 2 from each other in a hexagonal grid and in an octagonal grid. Hence, these systems are t-in-L diagnosable for $t = 4$ and $t = 5$, respectively.

In addition, for a hexagonal grid we can construct a syndrome and two allowable fault sets F_1 and F_2 such that for each $F_i, i = 1,2$, there exists a processor u with five faulty processors in $L(u) \cup \{u\}$.

Similarly, for an octagonal grid we can construct a syndrome and two allowable fault sets F_1 and F_2 such that for each $F_i, i = 1,2$, there exists a processor u with six faulty processors in $L(u) \cup \{u\}$.

Thus, the maximum values of t for which a hexagonal grid and an octagonal grid are t-in-L diagnosable are 4 and 5, respectively. □

V. t-in-L Diagnosis

In this section we consider t-in-L diagnosis. The following lemma forms the basis of our approach.

Lemma 4: Given a system S and a syndrome, let u be a processor in S such that $|L(u)| = k$, $L(u) \cup \{u\}$ has at most $\lfloor k/2 \rfloor + 1$ faulty processors, and at least two processors in $L(u)$ have been identified correctly. Then u can be identified correctly.

Proof: Let F denote the set of processors in $L(u)$ that have been identified correctly. If any member of F is fault-free then the status of u can be determined correctly. We now consider the case when all processors in F have been identified to be faulty. Let X_0 and X_1 represent the set of processors in $L(u) - F$ that has 0-links and 1-links, respectively, with u. If

$$|F| + |X_1| > \lfloor k/2 \rfloor + 1 \quad (1)$$

then u can be declared faulty; u can be declared fault-free if

$$|F| + |X_0| + 1 > \lfloor k/2 \rfloor + 1. \quad (2)$$

Both (1) and (2) cannot be satisfied simultaneously; otherwise, the assumption that there are at most $\lfloor k/2 \rfloor + 1$ faults in $L(u) \cup \{u\}$ is violated or the processors in F have been identified incorrectly. At least one of (1) and (2) is satisfied if we ensure that

$$|F| + \max\{|X_0| + 1, |X_1|\} > \lfloor k/2 \rfloor + 1. \quad (3)$$

Since $|X_0| + |X_1| = k - |F|$, $\max\{|X_0| + 1, |X_1|\} \geq \lfloor (k - |F|)/2 \rfloor + 1$.

But

$$|F| + \lfloor (k - |F|)/2 \rfloor + 1 \geq \lfloor k/2 \rfloor + 2$$

if $|F| \geq 2$. Hence (3) is satisfied if $|F| \geq 2$. □

Next we present a procedure called LABEL. This procedure is applicable to all the regular systems considered in the previous section as well as those that satisfy the conditions of Theorem 1. Given a fault-free processor, ν LABEL(v) determines the status of all the processors in the system.

procedure LABEL (v : node)

S.1) Label node v fault-free. Let A := {v}.

S.2) (a) Pick a node $x \notin$ A such that x is adjacent to a node in A and satisfies one of the following properties:

i. x is adjacent to a fault-free node y in A; (ii) x is adjacent to two faulty nodes in A; (iii) x is adjacent to a faulty node y in A which already has $\lfloor deg(y)/2 \rfloor$ nodes labeled faulty in $L(y)$.

(b) if (i) is true then label x as fault-free if x and y share a 0-link; label x as faulty, otherwise;
elseif (ii) is true then determine the label of x using Lemma 4;
elseif (iii) is true then lable x as fault-free.

(c) Add x to the set A.

S.3) Repeat S.2 until A = U.

end procedure

Note that it can be proved that for all the systems to which LABEL(v) is applicable, there exists a processor that satisfies one of the properties mentioned in S.2 of procedure LABEL. Thus in these cases the procedure will terminate after determining the status of all the processors.

Our approach to t-in-L diagnosis is as follows:

1) determine a fault-free processor ν;
2) apply procedure LABEL(v) to determine the status of all the other processors.

We now show that by applying LABEL(v) at most two times we can determine a fault-free processor.

First, we pick a node q with at least $\lfloor deg(q)/2 \rfloor - 1$ 0-links. Such a node exists since each fault-free processor has this property. In fact, a node with this property can be found in the neighborhood $L(u) \cup \{u\}$ of every processor u in the system. If the degree of $q \geq 3$, then let w and z be two nodes sharing 1-links with q; if q does not have two 1-links, then q must be fault-free. If the degree of q is two, w and z will be the two nodes adjacent to q.

Having selected w and z as earlier, we apply procedure LABEL on these two nodes. If either one of them determines a consistent labeling, then it is fault-free and we are finished. If both are faulty, then q must be fault-free; otherwise, $L(q) \cup \{q\}$ will have more than $\lfloor deg(q)/2 \rfloor + 1$ faulty processors, contradicting the local fault constraints.

Thus, we need to use procedure LABEL at most two times to determine a fault-free processor. One more application of this procedure on the fault-free processor will complete the diagnosis.

The complexity of our t-in-L diagnosis algorithm is dominated by the complexity of procedure LABEL, which is called at most three times. It can be shown that the complexity of LABEL(v) is $O(n^2)$. So the overall complexity of our diagnosis algorithm is also $O(n^2)$.

Summarizing our discussions, we have the following.

Theorem 5: Let S be a closed rectangular grid, a closed hexagonal grid, a closed octagonal grid system, or a hypercube system (with 2^p processors) in which for every $u_i \in U$ there are at most $\lfloor k/2 \rfloor + 1$ faulty processors in $L(u_i) \cup \{u_i\}$ where $k = 4, 6, 8$, and p, respectively. Given a syndrome, all processors in S can be identified correctly provided less than half the total number of processors in S are faulty. □

Theorem 6: Consider a system S with test interconnection graph $G = (U, E)$ in which for every $u_i \in U$, there are at most $\lfloor k/2 \rfloor + 1$ faulty processors in $L(u_i) \cup \{u_i\}$ where k is the degree of u_i. Let $|L(u_i)| \geq 3$ for all $u_i \in U$. Then given that less than $|U|/2$ processors in S are faulty, all processors in the system can be identified correctly in $O(n^2)$ time, if for any two processors x_i and x_j at distance 2 from each other in S, at least one of the following holds:

1) There are at least two vertex disjoint paths of length 2 between x_i and x_j.
2) The graph shown in Fig. 1(a) containing x_i and x_j is a subgraph of G.

VI. Summary and Conclusions

In this paper we have studied the problem of diagnosing faulty processors in a multiprocessor system when fault constraints in a local domain of each processor are specified. We have introduced the t-in-L diagnosability theory. A system S is t-in- L diagnosable if, given a syndrome, all faulty processors can be identified uniquely, provided there are at most t faulty processors in the local domain $L(u) \cup \{u\}$ of every processor in S. Assuming that less than half the processors in the system are faulty, we have shown that regular interconnected systems such as the hypercube systems and the closed rectangular, hexagonal, and octagonal grid systems are t-in-L diagnosable for $t = \lfloor \delta/2 \rfloor + 1$, where δ is the minimum degree of the interconnection graph. We have established a sufficient condition for a system to be t-in-L diagnosable for $t = \lfloor \delta/2 \rfloor + 1$. We have also presented t-in-L diagnosis algorithms for all the cases considered. These algorithms are of linear complexity with respect to the number of procesors in the system.

In most useful multiprocessor systems, each processor has direct connections to a small number of processors. If only processors with direct connections are allowed to test one another, then for most practical systems that are sparsely connected, the classical diagnosability theory will allow only a small number of faulty processors. The t-in-L diagnosability theory overcomes this shortcoming of the classical diagnosis approach. Our diagnosis algorithms can be implemented in a totally distributed manner on the system itself requiring no global syndrome analysis. Synchronous implementations of these diagnosis algorithms with linear message and time complexities (with respect to system size) can easily be designed.

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Geometrical Learning Algorithm for Multilayer Neural Networks in a Binary Field

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***Abstract*— This correspondence introduces a geometrical expansion learning algorithm for multilayer neural networks using unipolar binary neurons with integer connection weights, which guarantees convergence for any Boolean function. Neurons in the hidden layer develop as necessary without supervision. In addition, the computational amount is much less than that of the backpropagation algorithm.**

***Index Terms*—Binary field, convergence, hardlimiting neurons, integer weights, learning, neural networks.**

I. Introduction

Since the perceptron was proven to be incapable of classifying linearly inseparable patterns and pessimistically abandoned in 1960, there have been several technological breakthroughs such as the Hopfield neural network [1] and the backpropagation algorithm (BPA) [2]. In addition, the area has been prolific despite the many unsolved problems and inefficiencies. In addition to the slow learning speed of BPA, it has several other problems. First, the convergence of learning is not guaranteed in advance. In addition, the minimum structure of a backpropagation network (BPN) (or multilayer perceptron with sigmoid neurons) for a set of training patterns is not well understood. Moreover, for functions in discrete space, BPA searches weights and thresholds in a continuum space. Because of the unnecessary complexity of BPA, it usually requires hundreds of iterations to train a BPN even for very simple two-variable Boolean functions. In addition, practical hardware implementation of a BPN and BPA with a fair accuracy seems still unrealistic [3].

In this paper, a geometrical learning algorithm is introduced in an effort to resolve the problems mentioned earlier, especially for arbitrary functions in a binary field. Systematically finding a network using unipolar binary neurons and integer connection weights for an arbitrary Boolean function without using an ad hoc method is still an unsolved task even for a small number of input variables [4], [5]. The structure of networks for the functions presented herein is identical with that of the multilayer perceptron. However, the networks use neurons with a hard-limiting activation function and integer weights.

Moreover, one of the significant differences between BPA and the new learning algorithm is that the new one first finds the required hyperplanes based on a geometrical analysis of given patterns. It then finds the weights and thresholds based on these identified hyperplanes. However, BPA indirectly finds the hyperplanes by minimizing the error between the actual outputs and desired outputs [2].

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