On Dynamic Reconfiguration of Multi-Robot Formations

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Motivation

- Small, low cost expendable vehicles are becoming more common. They need to operate cooperatively in tactical situations;
- Many biological systems exhibit formation, coordination and group behaviors.

- Develop the theory and software tools to coordinate networks of semi-autonomous vehicles.

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Applications

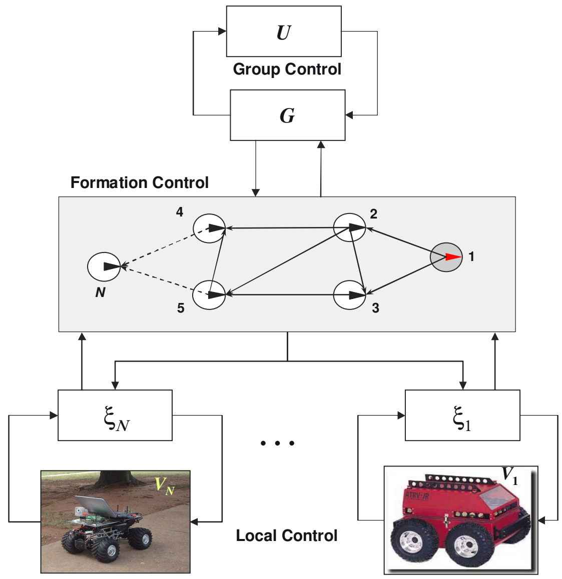
- Collaborative mapping and exploration
- Cooperative transport
- Force multiplication
- Relay communications
- Networks of smart mobile sensors
- Satellite clustering
- Swarming
- Formation Flight

Research Initiatives

RoboCup (Robots Playing Soccer)

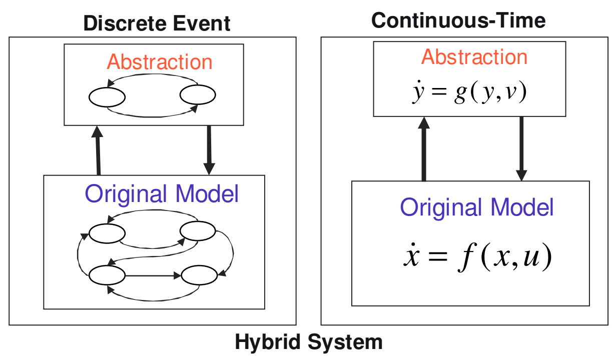
USAR (Urban Search and Rescue)

Hierarchical Architecture



Group Control

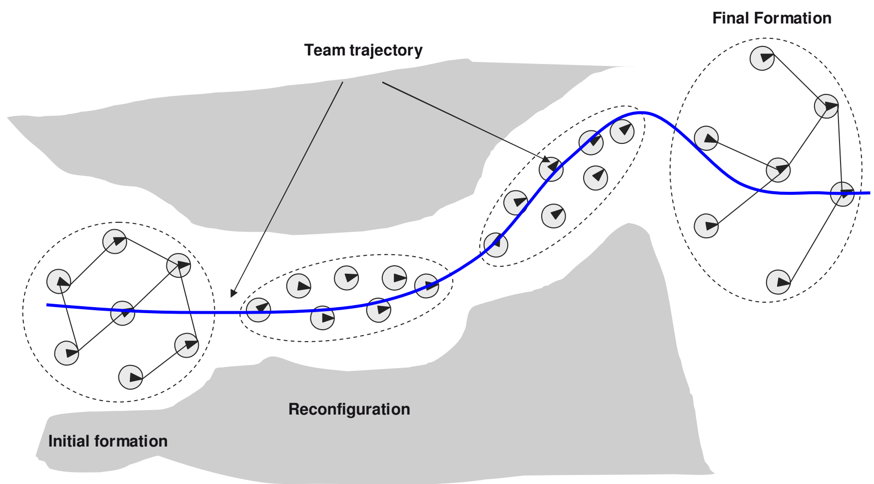
Hybrid Systems and Abstractions



- **Consistent abstractions**
- **Parallel and sequential composition**
- **Refinement**

Pappas *et al*, IEEE TRA, 2001

Example



Group Control: Abstractions

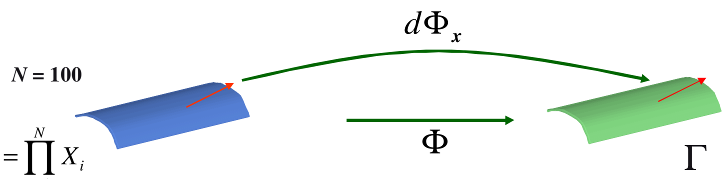
Develop suitable descriptions of the group of vehicles

- **Smaller dimensional system that describes team behavior,**
- **Preserve some properties of interest,**

$$\boldsymbol{x} = \{x_1, \ldots, x_N\} \in \prod_{i=1}^{N} X_i \equiv \boldsymbol{X} \subseteq \mathbb{R}^m \qquad \dot{x} = f(x, u)$$

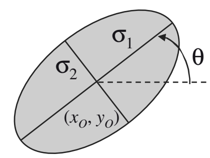
$$\dot{y} = g(y, v) \quad \text{with} \quad y \in \Gamma \subseteq \mathbb{R}^p \quad \text{and} \quad p \ll m$$

- **Smooth map** $\Phi : X \to \Gamma$ and $y = \Phi(x)$



$$X = \prod_{i=1}^{N} X_i \qquad y = \begin{bmatrix} x_O \\ y_O \\ \theta \\ \sigma_1 \\ \sigma_2 \end{bmatrix} \in R^5$$

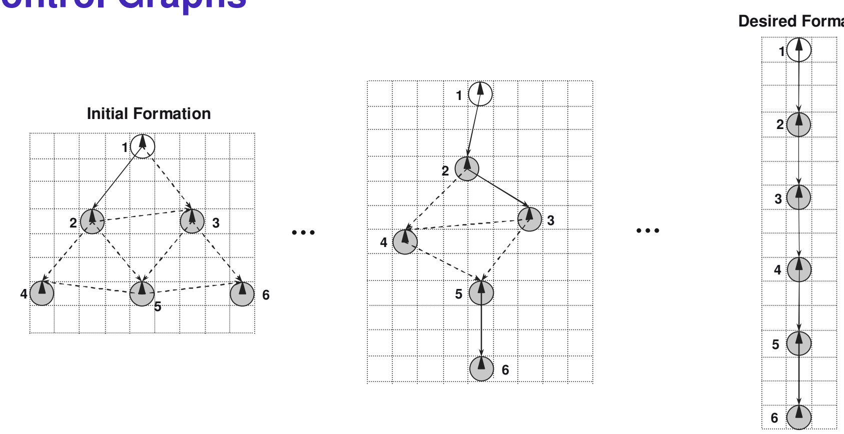
$$\dot{y} = \begin{bmatrix} \dot{x}_O \\ \dot{y}_O \\ \dot{\theta} \\ \dot{\sigma}_1 \\ \dot{\sigma}_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_O^d + k_x(x_O^d - x_O) \\ \dot{y}_O^d + k_y(y_O^d - y_O) \\ \dot{\theta}^d + k_\theta(\theta^d - \theta) \\ \dot{\sigma}_1^d + k_{\sigma_1}(\sigma_1^d - \sigma_1) \\ \dot{\sigma}_2^d + k_{\sigma_2}(\sigma_2^d - \sigma_2) \end{bmatrix}$$



work in progress...

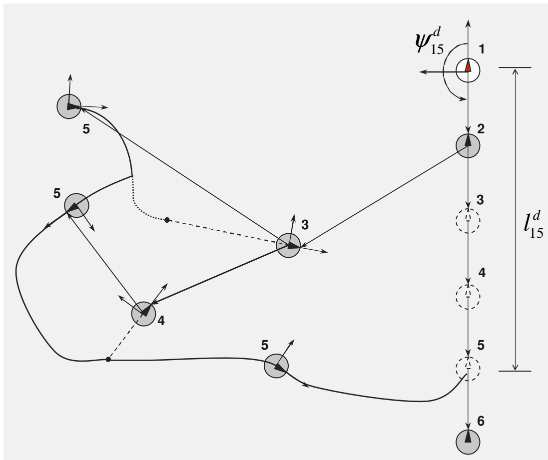
Optimization Based Formation Control

Control Graphs



- **Graph assignment algorithm**
- **Stability of the switched system becomes an issue**

Formation Shape S



- **Formation error**

$$\tilde{S} = \left\| \mathbf{r}^d - \mathbf{r} \right\|$$

- **Lyapunov-like function**

$$V = \tfrac{1}{2} \tilde{S}^2$$

- **The control graph is a dynamical systems that defines a switching sequence C_w**

The control graph should be assigned such that

$$\dot{V} \leq 0$$

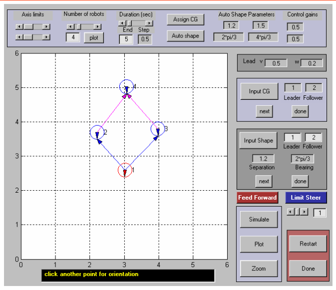
Fierro et al., ICRA2001

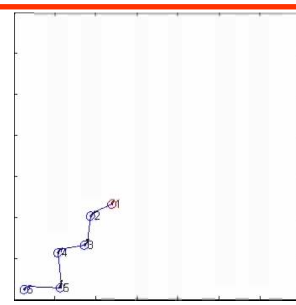
Graph Assignment Algorithm

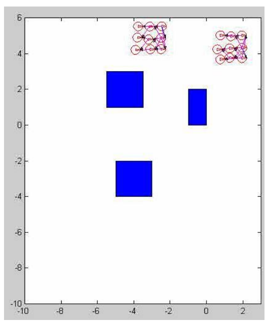
Algorithm 1 Control graph assignment algorithm (CGA)

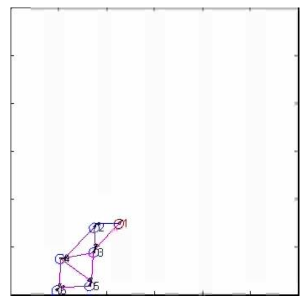
initialize adjacency matrix $H(i,j) := 0$;
for all robot $k \in \{1, 2, \ldots, n\}$, $k \neq leader$ **do**
  $H(i,k)$:=1 for $SB_{ik}C$, edges$(i,k) \in$ spanning tree of $\mathcal{G}_{vis}$;
  d_k:= depth of node k in communication graph $\mathcal{G}_{comm}$;
  find set P_k of robots visible to k with depths $d_k, d_k - 1$;
  if $P_k = \emptyset$(disconnected) **then**
    report failure at k, break;
  S_k:=P_k sorted by ascending $\delta t_{ik} K_{ik} \tilde{\mathcal{S}}_{ik}^2$ $(i \in P_k)$;
  if $numOfElements(S_k) \geq 2$ **then**
    pick last two elements $i, j \in P_k$;
    if $\epsilon_{ijk} = (l_{ik} + l_{jk} - l_{ij}) \neq 0$ **then**
      $H(i,k)$:= 1, $H(j,k)$:= 1 for $SS_{ijk}C$;
    else
      repeat above check for remaining $j \in S_k$ in order;
  generate set-points $\boldsymbol{r}^d$ for desired shape $\mathcal{S}^d$

Control Graph Assignment Algorithm



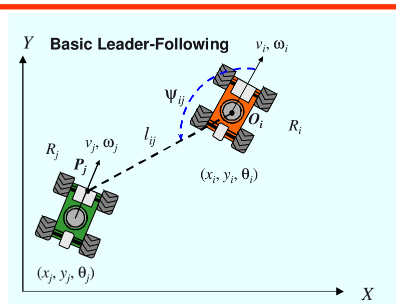


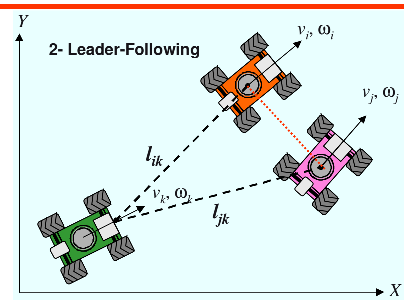


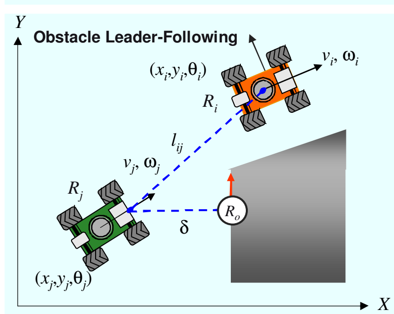


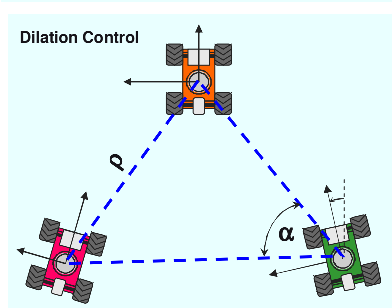
A. Das, and P. Song

Local (kinematic) Control

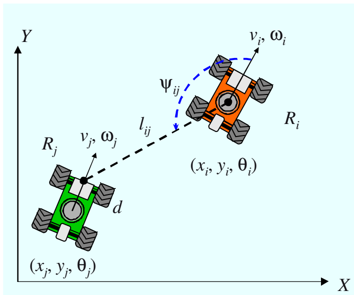








Basic Leader-Following



$$\dot{x}_i = v_i \cos\theta_i, \quad \dot{y}_i = v_i \sin\theta_i, \quad \dot{\theta}_i = \omega_i$$

Output vector: $z_j = \begin{bmatrix} l_{ij} \\ \psi_{ij} \end{bmatrix}$

$$\dot{z}_j = G_1(z_j)u_j + F_1(z_j, u_i),$$
$$\dot{\theta}_j = \omega_j$$

$$u_j = G_1^{-1}[k(z_j^d - z_j) - F_1]$$

$\Rightarrow$

$$\dot{l}_{ij} = k_1(l_{ij}^d - l_{ij})$$
$$\dot{\psi}_{ij} = k_2(\psi_{ij}^d - \psi_{ij})$$
$$\dot{\theta}_j = \omega_j$$

- **Theorem** Assume:
  - The leader's trajectory is well-behaved

  Then, the formation is stable and the system error of the linearized system converges exponentially to zero.

- **Remarks** Are the internal dynamics stable?

$$\dot{e}_\theta = -\frac{v_i}{d}\sin e_\theta + \eta_1(e_\theta, \omega_i, e_z)$$

Dual-Mode Model Predictive Control

MPC methods have some potential advantages over Input/Output feedback linearization approaches.

- Ability of incorporate constraints,
- Optimization-based methods may be more robust and more flexible in meeting performance requirements, but may be computationally expensive,
- We develop a dual-mode MPC algorithm with a terminal constraint instead of an MPC algorithm with a terminal cost,
- The optimization-based controller (MPC) drives the system to the terminal constraint set $\mathcal{X}_f$,
- Within $\mathcal{X}_f$ the system utilizes an I/O feedback linearization control law. Thus, stability can be proven.

Objective Function

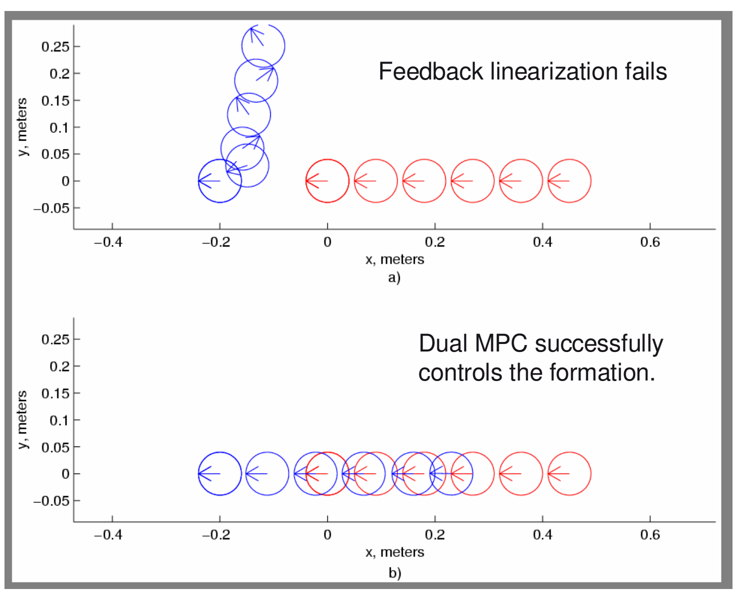
$$V(k) = V_{pos}(k) + V_{in}(k) + V_{col}(k)$$

$$V_{pos}(k) = \sum_{m=1}^{H_p} x^T(k+m) Q (k+m)$$

$$V_{in}(k) = \sum_{m=1}^{H_p} \Delta U^T(k-1+m) R \Delta U(k-1+m)$$

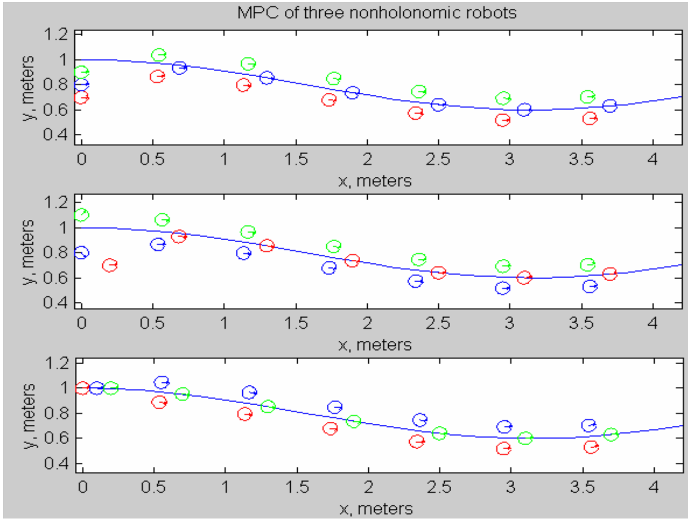
$$V_{col}(k) = \sum_{m=1}^{H_p} e^{-c_{ij}(k-1+m)/\tau}$$

$$c_{ij} = \left\| x_i - x_j \right\|_2 - r_{\min}$$



Bow, Port and Starboard

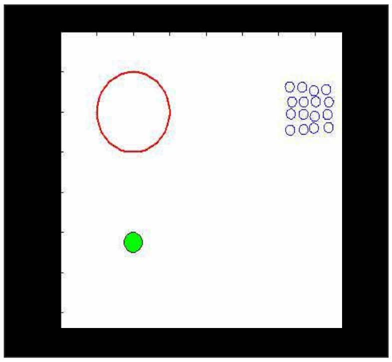
- We use nautical terms to emphasize the lack of leaders and followers

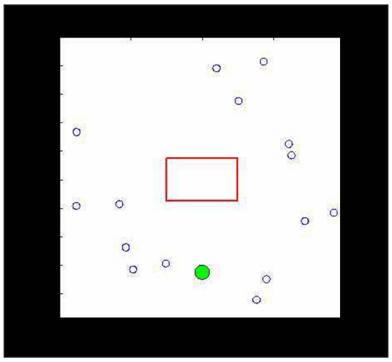


Applications

Coordinated manipulation:

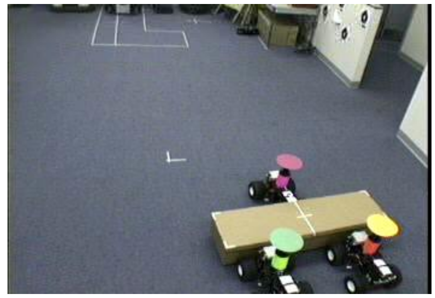
- Approach
- Organize
- Transport



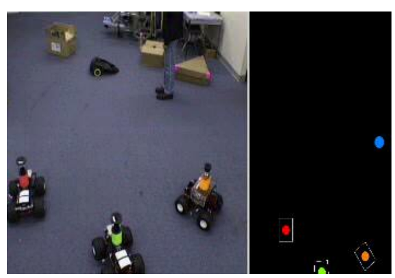


Multimedia Version of

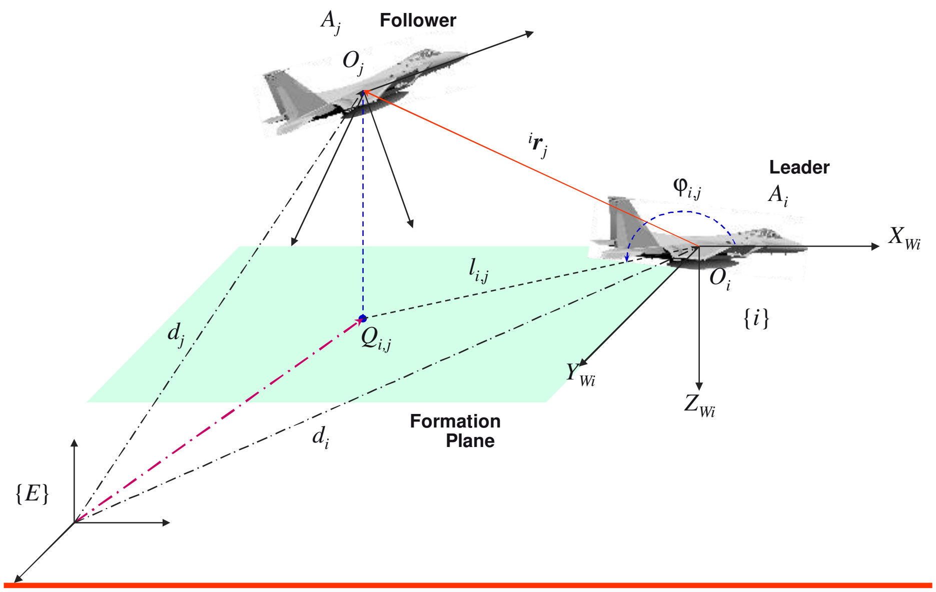
R. Fierro, A. Das, J. Spletzer, R. Alur, J. Esposito, Y. Hur, G. Grudic, V. Kumar, I. Lee, J. P. Ostrowski, G. Pappas, J. Southall and C. J. Taylor, "A Framework and Architecture for Multirobot Coordination," *International Journal of Robotics Research* (*IJRR*), vol. 21, no. 10-11, pp. 977-995, Oct-Nov. 2002.



Multimedia Version 2



Formation Flight Geometry



UAV Formation Control

- **The state vector is arranged into four sets**

$$x_1 = (V, \gamma, \chi)$$
$$x_2 = (\mu, \alpha, \beta)$$
$$x_3 = (p, q, r)$$
$$x_4 = ({}^E x, {}^E y, {}^E z)$$

$$u_1 = \delta_p$$
$$u_2 = (\delta_a, \delta_e, \delta_r)$$

- **The output vector is** $z_{ij} = \Big[\underbrace{{}^i x_j \quad {}^i y_j \quad {}^i z_j}_{\bar{x}_{4ij}} \quad \mu_{ij}\Big]^T$
  where

$$\begin{pmatrix} {}^i x_j \\ {}^i y_j \\ {}^i z_j \\ 1 \end{pmatrix} = {}^i A_E \begin{pmatrix} {}^E x_j \\ {}^E y_j \\ {}^E z_j \\ 1 \end{pmatrix}$$

$$\dot{\bar{x}}_{4ij} = F_4(x_{1_j}, X_i)$$

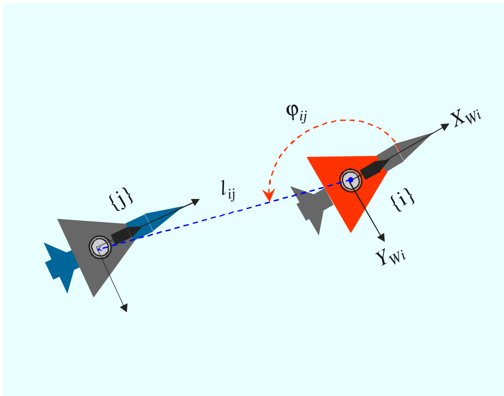
$$\mu_{ij} = \mu_i - \mu_j$$

We adopted A. Isidori's formulation

- **Applying I/O feedback linearization via dynamic extension**

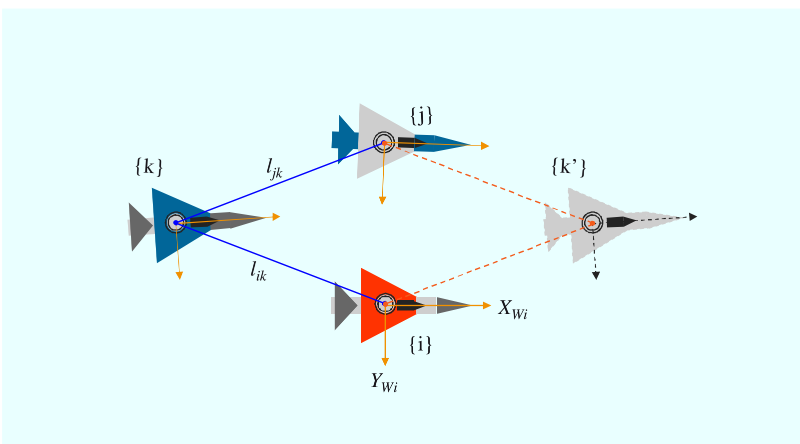
$${}^{i}x_j^{(4)} \equiv z_{1_{ij}}^{(4)} = \overline{w}_1 \qquad {}^{i}y_j^{(4)} \equiv z_{2_{ij}}^{(4)} = \overline{w}_2 \qquad {}^{i}z_j^{(4)} \equiv z_{3_{ij}}^{(4)} = \overline{w}_3 \qquad \ddot{\mu}_{ij} \equiv \ddot{z}_{4_{ij}} = \overline{w}_4$$

Basic Leader-Following



$$z_{1_{(i,j)k}}^{(4)} = \overline{w}_{1k} \qquad z_{2_{(i,j)k}}^{(4)} = \overline{w}_{2k} \qquad z_{3_{(i,j)k}}^{(4)} = \overline{w}_{3k} \qquad \ddot{\mu}_{ik} \equiv \ddot{z}_{4_{ik}} = \overline{w}_{4k}$$

2- Leader-Following



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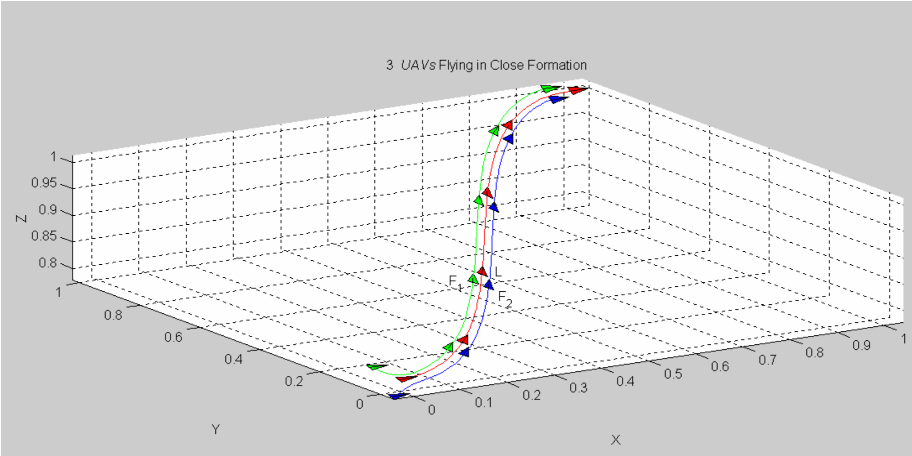
Simulation Results



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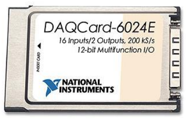
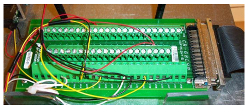
Multi-Robot Experimental Testbed at OSU

- Off-the-shelf components
- Tamiya^{TM} TXT-1 Platform

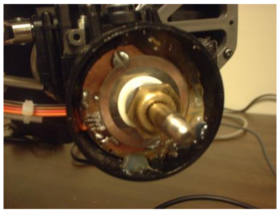


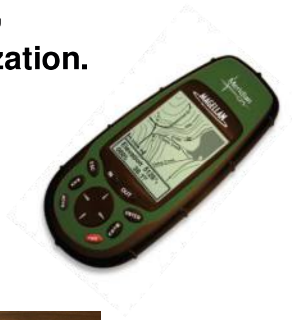
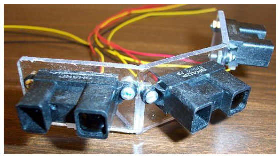


- Laptops (< 2.7 lb.)
- Integrated wireless,
- Firewire port,
- USB, etc.
- Serial servo controller

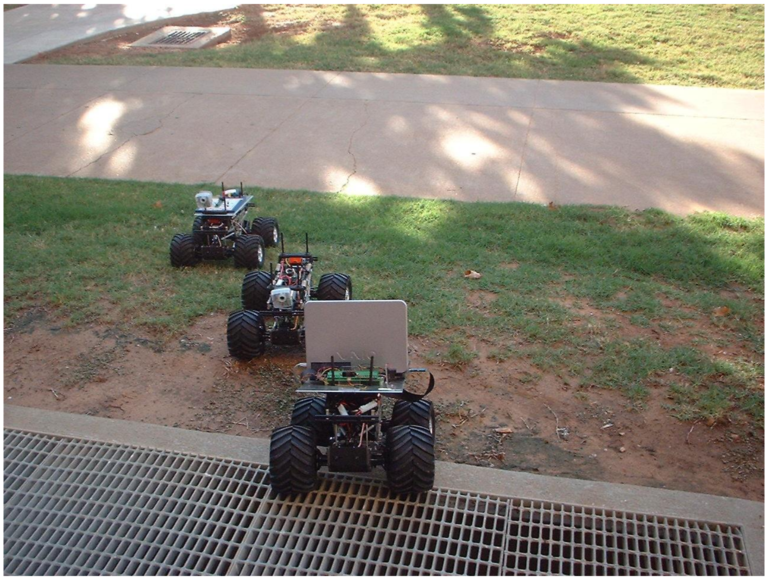
Sensors

- Stereo vision for cooperative sensing, exploration and mapping,
- IR and sonar for obstacle avoidance,
- GPS, odometer, and gyros for localization.




Multi-Robot Experimental Testbed at OSU



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Conclusions and Future Work

- We describe a modular architecture for coordinating distributed multi-vehicle systems;
- We present two optimization-based control approaches;
- Work has to be done at the interfaces of sensing, communication and control;
- Learning should be integrated at all levels.
- Need for a research agenda:
  - Define metrics for cooperative control;
  - Implement and evaluate different communication architectures, and networking technologies;
  - Vision based-control of UAVs.

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Relevant Publications

R. Fierro, A. Das, J. Spletzer, R. Alur, J. Esposito, Y. Hur, G. Grudic, V. Kumar, I. Lee, J. P. Ostrowski, G. Pappas, J. Southall and C. J. Taylor, "A Framework and Architecture for Multirobot Coordination," *International Journal of Robotics Research* (*IJRR*), vol. 21, no. 10-11, pp. 977-995, Oct-Nov. 2002.

R. Fierro, P. Song, A. Das, and V. Kumar, "Cooperative control of robot formations," in *Cooperative Control and Optimization*, R. Murphey and P. Pardalos (eds.), Applied Optimization, vol. 66, chapter 5, pp. 73-93, Kluwer Academic Press, 2002.

A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, "A framework for vision based formation control," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 813-825, Oct. 2002.

R. Fierro, A. Das, V. Kumar, and J. P. Ostrowski, "Hybrid control of formations of robots," *IEEE Int. Conf. on Robotics and Automation*, Seoul, Korea, May 2001, pp. 157-162. Finalist in the Best Conference Paper Competition.

A. K. Das, R. Fierro, and V. Kumar, "Control graphs for robot networks," in *Cooperative Control and Optimization*, R. Murphey and P. Pardalos (eds.), Applied Optimization, chapter 4, pp. 55-73, Kluwer Academic Press, 2002. (In press)

R. Fierro and A. K. Das, "A modular architecture for formation control," *IEEE 3rd International Workshop on Robot Motion and Control* (RoMoCo'02), Bukowy Dworek, Poland, Nov. 9-11, 2002.

R. Fierro, C. Belta, J. Desai, and V. Kumar, "On controlling aircraft formations," *Proc. IEEE Conference on Decision and Control*, *CDC2001*, Orlando, FL, Dec. 2001, pp. 1065-1070.