DCC Linear Congruential Graphs: A New Class of Interconnection Networks

J. Opatrný, D. Sotteau, N. Srinivasan, and K. Thulasiraman, Fellow, IEEE

Abstract—Let \( n \) be an integer and \( F = \{ f_i : 1 \leq i \leq t \} \) be a finite set of linear functions. We define a linear congruential graph \( G(F, n) \) as a graph on the vertex set \( V = \{ 0, 1, \ldots, n - 1 \} \), in which any \( x \in V \) is adjacent to \( f_i(x) \bmod n, 1 \leq i \leq t \).

For a linear function \( g \), and a subset \( V_1 \) of \( V \) we define a linear congruential graph \( G(F, n, g, V_1) \) as a graph on vertex set \( V \), in which any \( x \in V \) is adjacent to \( f_i(x) \bmod n, 1 \leq i \leq t \), and any \( x \in V_1 \) is also adjacent to \( g(x) \bmod n \).

These graphs generalize several well-known families of graphs, e.g., the de Bruijn graphs. We give a family of linear functions, called DCC linear functions, that generate regular, highly connected graphs which are of substantially larger order than de Bruijn graphs of the same degree and diameter. Some theoretical and empirical properties of these graphs are given and their structural properties are studied.

Index Terms—Graph theory, interconnections networks, network design, parallel processing, computer networks.

1 INTRODUCTION

In the design of massively parallel computers, one of the most important problems is the design of the interconnection network connecting the processors of the parallel computer. As stated in Hillis [18], the topology of the interconnection network imposes many performance restrictions. Any interconnection network can be represented as a graph in which the vertices correspond to the processors and the edges to the communication links. Thus, some of the properties of interconnection networks have been investigated in a graph-theoretical setting. For graph theoretical notation and terminology we follow [10]. The number of vertices of a graph will be called the order of the graph.

A family of graphs \( \mathcal{F} \) that is suitable for interconnection networks should contain infinitely many graphs of different orders and degrees. Furthermore, graphs in \( \mathcal{F} \) should be regular and of small degree, of relatively small diameter, high connectivity, and extensible, i.e., it is possible to construct a large graph in \( \mathcal{F} \) from smaller graphs in the family. See, for example, [8], [18], [26] for more detailed discussions of these issues.

The problem of constructing large regular graphs of given degree \( d \) and diameter \( D \), called \((d, D)\) graph problem, and the related problem of constructing a network of given order \( n \) and degree \( d \) with smallest possible diameter, proposed first in [16], has attracted the attention of many researchers, see [3], [12] for surveys. The upper bound on the order \( n \) of a graph of degree \( d > 2 \) and diameter \( D \), called the Moore bound, is as follows:

\[
    n \leq (d(d - 1)^{D - 2})/(d - 2).
\]

For \( D > 2 \), and \( d > 2 \), the Moore bound cannot be attained [12]. Hence the main interest has been in constructing networks of degree \( d > 2 \) and diameter \( D > 2 \) whose order approaches Moore bound. See the tables in [12], [13] for the largest known graph orders for small values of \( d \) and \( D \). These largest known graphs are constructed by different methods for different degrees and diameters, and they do not necessarily have many of the properties required for interconnection networks. The best families of graphs that contain large graphs of low diameter and have good network properties are de Bruijn graphs [11], and some of their variations, such as Kautz graphs [21], generalized de Bruijn graphs [14], [15] and Imase-Itoh graphs [19]. De Bruijn graphs were investigated for their suitability for communication networks, initially by Pradhan and Samanthi in [27], [28], Bermond and Peyrat in [8], and in many other papers by now. It is shown in [27], [28] that a binary de Bruijn network can solve a wide variety of classes of problems. Graphs having more vertices than de Bruijn graphs of the same degree and diameter can be obtained by a subgraph substitution into de Bruijn graphs [20].

Akers and Krishnamurthy [1] developed a formal group-theoretic model, called the Cayley graph model, for designing symmetric interconnection networks. Given a set of generators for a finite group \( G \), the Cayley graph of \( G \) is the graph in which the vertices correspond to the elements of the group, and the edges correspond to the action of the generators. It is shown in [1] that two classes of these networks, called the star graphs and the pancake graphs satisfy many of the desirable network properties given above, and they can accommodate many more vertices than a hypercube of the same diameter and degree.

The order of graphs, as a function of the degree and the diameter, in the families of graphs mentioned above is poor...
comparing to the Moore bound and to the bound obtained from the studies of random graphs [9]. Hence further improvements are needed.

Taking an approach which, in spirit, follows the same thrust as Akers and Krishnamurthy in [1], Opatrný and Sotteau proposed in [25] Linear congruential graphs. In a linear congruential graph of order \( n \), the vertices are integers between 0 and \( n - 1 \) and the adjacencies are defined by a set of linear functions. These graphs generalize de Bruijn graphs and some other families of graphs.

In this paper we define a new subfamily of linear congruential graphs, called DCC linear congruential graphs (DCC for Disjoint Consecutive Cycles), and investigate in detail several properties of graphs in this class. DCC linear congruential graphs are highly connected, regular graphs which can be constructed for any fixed degree \( d \) and any order \( n \) such that \( n \) contains a multiple factor. These graphs are much larger than de Bruijn graphs of the same degree and diameter.

In Section 2 of this paper we give a definition of linear congruential graphs, and some sufficient conditions for linear congruential graphs to be regular. Furthermore, the class of DCC linear congruential graphs is introduced.

In Section 3 we discuss DCC linear congruential graphs of order \( 2^p \) for some integer \( p \). It is shown that DCC linear congruential graphs of even degree are maximally connected. Some structural properties of these graphs are discussed. An upper bound on the diameter of DCC linear congruential graphs of degree 4 in terms of their order is given. However, this bound is not close to the actual value of the diameters of generated graphs. We give tables of diameters of DCC graphs of various orders and degrees and give some conjectures concerning the diameter of DCC linear congruential graphs.

Since DCC graphs satisfy many of the requirements of interconnection networks stated above, they could be considered as an alternative for very large interconnection networks.

2 Linear Congruential Graphs

We use \( \mathbb{N} \) to denote the set of nonnegative integers.

Definition 2.1. Let \( n \) be a positive integer and \( F \) be a finite set of \( t \) linear functions for some integer \( t \), \( F = \{ f_i(x) = (a_i x + c_i) : 1 \leq i \leq t, a_i, c_i \in \mathbb{N} \} \). We define a linear congruential graph \( G(F, n) \) as the graph on the vertex set \( V = \{0, 1, \ldots, n - 1\} \), in which any \( x \in V \) is adjacent to \( f_i(x) \mod n \), for every \( i, 1 \leq i \leq t \). For a subset \( V_1 \) of \( V \) and a linear function \( g \), we define a linear congruential graph \( G(F, n, g, V_1) \) as the graph on vertex set \( V \), in which any \( x \in V \) is adjacent to \( f_i(x) \mod n \), for every \( i, 1 \leq i \leq t \), and any \( x \in V_1 \) is also adjacent to \( g(x) \mod n \).

We call the functions in \( F \) union \( g \), the generators of \( G(F, n) \) and \( G(F, n, g, V_1) \), respectively. For any linear function \( f \) we call the graph \( G(f, n) \) the graph generated by \( f \) on \( 0, 1, \ldots, n - 1 \). See Fig. 1 for an example of a linear congruential graph.

We will show that, for a suitable chosen set of generators \( F \), the graph \( G(F, n) \) is a regular graph of even degree. If \( n \) is even then for a suitable chosen set of generators \( F \cup \{ g \} \), the graph \( G(F, n, g, V_1) \) will be shown to be a regular graph of odd degree.

![Fig. 1](attachment:image.png)

Fig. 1. A linear congruential graph.

Clearly, \( G(F, n) \), (or \( G(F, n, g, V_1) \)), could be also considered as a directed graph in which there is an arc from any \( x \in V \) to \( f(x) \mod n \), for every \( i, 1 \leq i \leq t \), (or from any \( x \in V \) to \( f(x) \mod n \), for every \( i, 1 \leq i \leq t \), and from any vertex \( x \) of \( V \), to \( g(x) \)). In this paper we restrict our attention to undirected graphs.

Graphs with edge sets defined by linear functions modulo the order of the graph were investigated in [22] as possible expanders but no other of their properties was considered there.

The class of linear congruential graphs is a very broad family of graphs. By imposing some restrictions on the values of the constants of the generators, we can obtain subfamilies of linear congruential graphs. For example, if in a graph \( G(F, n) \) all multiplicative constants of the generators in \( F \) are equal to 1, and if one of the generators is the function \( x + 1 \), then we obtain a distributed loop graph [4]. The de Bruijn graph \( G(d, D) \) of degree \( d \) and diameter \( D \) is isomorphic to a linear congruential graph \( G(F, d') \) where \( F = \{ dx + i : 0 \leq i \leq d - 1 \} \). Similarly Kautz graphs and generalized de Bruijn graphs (see [2], [3], [21], [15]) can all be obtained as subfamilies of linear congruential graphs.

Our goal is to define a new subfamily of linear congruential graphs that would have very good network properties. We first study the structure of linear congruential graphs generated by single generators. The results of this study will be used to obtain sufficient conditions for a set of generators to generate regular graphs.

Let \( n \) be a positive integer, \( f(x) = ax + c \) be a linear function, where \( a, c \in \mathbb{N} \), and let \( x_0 \in \{0, 1, \ldots, n - 1\} \). The sequence of integers \( x_0, x_1, \ldots, x_\ell \ldots \), defined by \( x_i = f(x_{i-1}) \mod n \) for \( i > 0 \), called a linear congruential sequence [23], is a periodic sequence with period length less than or equal to \( n \). By the definition of the edge set of the linear congruential graphs, the elements of the sequence corresponding to a period of the linear congruential sequence form a cycle in the graph generated by \( f \) on \( 0, 1, \ldots, n - 1 \). We now state
two lemmas that will be needed in our paper. The first lemma gives the necessary and sufficient conditions on a linear function \( f \) and an integer \( n \) to define a linear congruential sequence of period length equal to \( n \). Its proof is given in detail in [23].

**Lemma 2.2.** [23] Let \( f(x) = ax + c \) be a linear function, \( n \) be a positive integer, and \( x \in \{0, 1, 2, \ldots, (n - 1)\} \). For a given \( x_0 \), the linear congruential sequence \( x_0, x_1, x_2, \ldots, x_p, \ldots \) defined for \( i \geq 1 \) by \( x_i = (ax_{i-1} + c) \mod n \), has a period of length \( n \) if and only if

1) \( \gcd(c, n) = 1 \),
2) \( b = (a - 1) \) is a multiple of every prime factor of \( n \); \( b \) is also a multiple of 4, if \( n \) is a multiple of 4.

If a linear function \( f \) satisfies the conditions of Lemma 2.2 with respect to \( n \) then the graph \( G(f, n) \) consists of a Hamiltonian cycle on \( (0, 1, \ldots, n - 1) \). This is the case, for example, when \( n \) is a power of 2 and \( f \) is a function from the set \( \{5x + 1, 5x + 3, \ldots, 9x + 3, \ldots, 17x + 1, \ldots\} \).

For a graph \( G(F, n) \), an interesting case arises when each linear function in \( F \) satisfies the conditions of Lemma 2.2 with respect to \( n \), and \( G(F, n) \) thus consists of \( |F| \) Hamiltonian cycles (not necessarily edge disjoint without additional conditions on the constants in the functions). Linear congruential sequences with maximum period lengths are used in pseudorandom number generators. Since random graphs are almost always of low diameter [9], it was expected that \( G(F, n) \) graphs consisting of edge disjoint Hamiltonian cycles corresponding to linear congruential sequences of period length \( n \) could be graphs of low diameter. Such graphs were considered in [25], where many graphs whose orders are much larger than the orders of de Bruijn graphs of the same degree and diameter were constructed. One difficulty encountered in that case is the choice of the constants of the linear functions in \( F \) so that any two functions in \( F \) define edge-disjoint Hamiltonian cycles, which would produce a regular graph. Our subsequent investigations indicated that graphs with smaller diameter can be obtained when only one of the functions in \( F \) generates a Hamiltonian cycle, and all other functions generate several cycles on \( (0, 1, \ldots, n - 1) \). This is the case considered in this paper.

Lemma 2.3 gives sufficient conditions on a linear function to generate several cycles of equal length on the vertex set \( \{0, 1, \ldots, n - 1\} \).

**Lemma 2.3.** Let \( n \) be a positive integer that contains at least one multiple factor, i.e., \( n = k^p \cdot m \) for some integers \( k > 1 \), \( p \geq 2 \), and \( m \). Let \( c \) be an integer such that \( \gcd(c, n) = 1 \). Let \( b \) be a multiple of every prime factor of \( n \); \( b \) is also a multiple of 4, if \( n \) is a multiple of 4. For any \( i, 1 \leq i \leq p + 1 \), let \( f_i(x) = (k^p \cdot b + 1) x + k^i \cdot c \).

Then, the function \( f_i \) generates \( k^{i-1} \) vertex-disjoint cycles of length \( \frac{n}{k^i} \) on the set \( \{0, 1, \ldots, n - 1\} \). The vertex sets of these cycles are the sets

\[
A_{0i} = \{0, k^{i-1}, 2k^{i-1}, \ldots, n - k^{i-1}\},
A_{1i} = \{1, k^{i-1} + 1, \ldots, n - k^{i-1} + 1\}, \ldots,
A_{k^{i-1}-1} = \{k^{i-1} - 1, 2k^{i-1} - 1, \ldots, n - 1\}.
\]

Furthermore, there is an edge between two vertices \( x \) and \( y \) in the graph generated by \( f_i \) only if \( |y - x| \) is divisible by \( k^i \) but not by \( k^j \).

**Proof.** Let \( i \) be an integer, \( 1 \leq i \leq p + 1 \), and \( r \) be an integer, \( 0 \leq r \leq k^{i-1} - 1 \). Consider any integer \( k^{i-3} q + r \) of the set \( A_{(r+1)i} \). Then

\[
f_i(k^{i-3} q + r) \mod n = \frac{(k^{i-1}b + 1)(k^{i-1}q + r) + k^i c) \mod k^i m}
\]

\[
= (k^{i-1}(k^{i-1}b + 1)q + k^i c) \mod k^i m + r
\]

\[
= k^i \left[ (k^{i-1}b + 1)q + (br + c) \right] \mod k^{i+1} m + r.
\]

Thus \( f_i(k^{i-3} q + r) \mod n \) also belongs to the same set \( A_{(r+1)i} \).

Now, let us consider the linear function \( h(x) = (k^{i-1}b + 1)x + (br + c) \). Since \( b \) is divisible by any prime factor of \( n \), and \( \gcd(b, n) = 1 \), then \( \gcd(br + c, k^{i+1} m) = 1 \) and so \( k^{i-1}b + 1 \) satisfies condition 1) of Lemma 2.2 with respect to \( k^{i+1} m \). Also \( h(x) \) satisfies condition 2) of Lemma 2.2 with respect to \( k^{i+1} m \). Hence, by Lemma 2.2, the linear congruence \( h(x) \mod k^{i+1} m \) generates a linear congruential sequence of period \( k^{i+1} m \) on the set \( \{0, 1, \ldots, k^{i+1} m - 1\} \).

This implies that, for every \( r, 0 \leq r \leq k^{i-1} - 1 \), the function \( f_i \) generates a cycle of length \( k^{i+1} m \) on the set \( A_{(r+1)i} \).

If \( (x, y) \) is an edge in the graph generated by \( f_i \) then we have

\[
|y - x| = |f_i(x) \mod n - x|
\]

\[
= |(k^{i-1}b + 1)x + k^i c - sn - x|
\]

\[
= k^{i-1}bx + k^{i-1}c - sn
\]

\[
= k^{i-1}(bx + c - sk^{i+1}m)
\]

for some integer \( s \). Since \( b \) is divisible by \( k \) and \( c \) is not divisible by \( k \), \( |y - x| \) is divisible by \( k^{i-1} \) but not by \( k^i \).

A linear function defined as in Lemma 2.3, which generates \( k^i \) disjoint cycles, will be called a cycle type \( k^i \) on the set \( \{0, 1, \ldots, n - 1\} \). Similarly a function satisfying Lemma 2.2 which generates a Hamiltonian cycle will be called of cycle type 1 on the set \( \{0, 1, \ldots, n - 1\} \).

For example, function \( 9x + 2 \) is of cycle type 2 and 17x + 4 is of cycle type 4 on the set \( \{0, 1, \ldots, 16\} \) for \( p \geq 2 \).

In the lemma above, if \( m \) is equal to 1 or 2 and \( i \) is equal to \( p + 1 \), or if \( k \) is equal to 2, is equal to 1, and \( i \) is equal to \( p \), then the generator \( f_i \) generates loops, or multiple edges in the graph. Since we are interested in simple graphs without loops and multiple edges, in our constructions we only use generators that give cycles of lengths larger than 2. We can achieve this by stipulating that any generator generates at most \( k^i \) cycles with \( k^i < \frac{n}{k^i} \).

**Theorem 2.4.** Let \( n \) be a positive integer that contains at least one multiple factor, i.e., \( n = k^p \cdot m \) for some integers \( k > 1 \), \( p \geq 2 \),
and m. For an integer t in \([0, 1, \ldots, p + 1]\), let \(F\) be a set of \(t\) linear functions such that:

1) each function in \(F\) is of cycle type \(k^j\) on \([0, 1, \ldots, n - 1]\) for some \(j\), \(0 \leq j \leq p\), such that \(k^j < \frac{n}{q}\);
2) there is exactly one function in \(F\) of cycle type 1 on the set \([0, 1, \ldots, n - 1]\);
3) any two functions in \(F\) are of different cycle types on \([0, 1, \ldots, n - 1]\).

Then the graph \(G(F, n)\) is a regular, connected graph of degree \(2t\).

If \(k = 2\), and for some linear function \(g\) the set \(F \cup \{g\}\) satisfies the three conditions above, and \(g\) is of cycle type \(2^j\) on \([0, 1, \ldots, n - 1]\) for some \(l \leq 1\), then the graph \(G(F, n, g, V_1)\), where \(V_1 = \{0, 1, \ldots, 2^j - 1, 2^{j+1}, 2^{j+1} + 1, \ldots, 2^{j+k} - 1, \ldots\}\), is a regular, connected graph of degree \(2t + 1\).

**Proof.** Since \(F\) includes a function that generates a Hamiltonian cycle, the graphs \(G(F, n)\) and \(G(F, n, g, V_1)\) are connected. Since any two functions in \(F\) or \(F \cup \{g\}\) are of different cycle types with respect to \(n\), the cycles generated by these two functions are edge disjoint by Lemma 2.3. Thus each function in \(F\) contributes 2 to the degree of each vertex and the graph \(G(F, n, g, V_1)\) is of degree \(2t\).

We have chosen \(V_1\) so that the graph \(G(F, n, g, V_1)\) contains every second edge of the cycles generated by \(g\) on the set \([0, 1, \ldots, n - 1]\). Since \(k = 2\), each cycle is of even length by Lemma 2.3 and thus \(g\) restricted to \(V_1\) generates a perfect matching in the graph. Therefore, \(G(F, n, g, V_1)\) is a regular graph of degree \(2t + 1\).

In our experiments the best results, with respect to the diameter of linear congruential graphs of order \(n = km\) and degree \(2t + 1\), have been obtained when the generators satisfy the conditions of Theorem 2.4 and, furthermore, when the set \(F\) or \(F \cup \{g\}\) contains a function of cycle type \(k^j\) for each \(j, 0 \leq j \leq t - 1\) or \(j, 0 \leq j \leq t\). With this in view we introduce the notion of a **Disjoint Consecutive Cycles set of generators**.

**Definition 2.5.** Let \(n\) be a positive integer that contains at least one multiple factor, i.e., \(n = km\) for some integers \(k > 1, p \geq 2\), and \(m\). For any given \(t \in \{1, \ldots, p + 1\}\), such that \(k^{-1} < \frac{1}{q}\), we say that a set \(F\) of \(t\) linear functions is a Disjoint Consecutive Cycles set (DCC set for short) with respect to the integer \(n\) if for each \(j, 0 \leq j \leq t - 1\), there is exactly one function in \(F\) of cycle type \(k^j\) on the set \([0, 1, \ldots, n - 1]\).

For example, \((5x + 1, 9x + 2, 17x + 4)\) and \((5x + 3, 9x + 10, 17x + 12)\) are DCC sets of generators, each of cycle type \(1, 2, 4\) with respect to \(2^2\) for \(p \geq 4\), while \((4x + 1, 10x + 3, 28x + 9)\) is a DCC set of generators of cycle types \(1, 3, 9\) with respect to \(3^2\) for \(p \geq 3\).

**Definition 2.6.** Let \(n\) be a positive integer that contains at least one multiple factor, i.e., \(n = km\) for some integers \(k > 1, p \geq 2\), and \(m\). For any given \(t \in \{1, \ldots, p + 1\}\) such that \(k^{-1} < \frac{1}{q}\), we define a DCC linear congruential graph \(G_d(F, n)\) as a linear congruential graph \(G_d(F, n)\) generated by a DCC set \(F\) of \(t\) linear functions with respect to \(n\). If \(k = 2, t \leq p, \) and \(g\) is a linear function of cycle type \(2^j\), we define a DCC linear congruential graph \(G_{2x+1}(F, n, g)\) as a linear congruential graph \(G_d(F, n, g, V_1)\), where

\[ V_1 = \{0, 1, \ldots, 2^j - 1, 2^{j+1}, 2^{j+1} + 1, \ldots, 2^{j+k} - 1, \ldots\} \]

Notice that the graph in Fig. 1 is a DCC linear congruential graph. Theorem 2.4 gives immediately the following result.

**Corollary 2.7.** Let \(n\) be a positive integer that contains at least one multiple factor, i.e., \(n = km\) for some integers \(k > 1, p \geq 2\), and \(m\). For any given \(t \in \{1, \ldots, p + 1\}\), such that \(k^{-1} < \frac{1}{q}\), let \(F\) be a DCC set of \(t\) linear functions. Then the graph \(G_d(F, n)\) is a regular, connected graph of degree \(2t\). If \(k = 2\) and \(t = p\) then the graph \(G_{2x+1}(F, n, g)\) is a regular, connected graph of degree \(2t + 1\).

A question could be asked whether our choice of set \(V_1\) in defining a DCC linear congruential graph of odd degree was good. Clearly, if the function \(g\) of cycle type \(2^j\) generates a perfect matching on the set \([0, 1, \ldots, n - 1]\) when the function is applied to the set \(V_1\), the function \(g\) would also generate a perfect matching when applied to the set

\[ V_2 = \{0, 1, \ldots, n - 1\} - V_1, \]

which could possibly produce better results. The following theorem disproves this possibility.

**Theorem 2.8.** Let \(F\) and \(F \cup \{g\}\) be DCC sets of linear functions with respect to \(n = 2m\) for some integer \(p > 1, \) and \(|F| = t\). Let

\[ V_1 = \{0, 1, \ldots, 2^j - 1, 2^{j+1}, 2^{j+1} + 1, \ldots, 2^{j+k} - 1, \ldots\}, \quad V_2 = \{0, 1, \ldots, n - 1\} - V_1. \]

There exists a linear function \(h\) such that the graph \(G_1 = G(F, n, g, V_1)\) is identical to the graph \(G_2 = G(F, n, h, V_2)\).

**Proof.** For any element \(x \in V_2\), \(g(x) \mod n\) is an element of the set \(V_1\). Since \(g\) is a one-to-one linear mapping of \([0, 1, \ldots, n - 1]\) onto \([0, 1, \ldots, n - 1]\), there exists a linear function \(h\) which is an inverse of \(g\) on the set \([0, 1, \ldots, n - 1]\). Therefore, the matching defined by \(h\) on \(V_1\) is identical to the matching defined by \(g\) on \(V_2\) and \(h\) is of the same cycle type as \(g\).

**3 Properties of DCC Linear Congruential Graphs**

In this section we will study in detail the DCC linear congruential graphs whose order \(n\) is a power of 2, i.e., \(n = 2^p\) for some integer \(p\). Thus, the generators considered in this section will be a DCC set of linear functions

\[ F = \{f_i : 1 \leq i \leq t\} \]

for some \(t \leq p - 2\), with

\[ f_i \in \{aq + c_i : q_i = 2^{j+1}b' + 1, c_i = 2^{j-1}(2r + 1) | b', r \in N\}. \]

These functions satisfy the conditions of Lemma 2.3 for \(i > 1\), and the conditions of Lemma 2.2 for \(i = 1\) with respect to the powers of 2. Recall that, for any \(i \in \{1, \ldots, p - 1\}\), the
function $f_i$ is of cycle type $2^{i-1}$ on the set $\{0, 1, ..., 2^i - 1\}$. Thus the $2^{i-1}$ disjoint cycles generated by $f_i$ partition the vertex set $\{0, 1, ..., 2^i - 1\}$ into $2^{i-1}$ disjoint subsets of cardinality $2^{i-1}$:

$$A_{1i} = \{0, 2^{i-1}, 2^i, ..., n - 2^{i-1}\},$$
$$A_{2i} = \{1, 2^{i-1} + 1, 2^i + 1, ..., n - 2^{i-1} + 1\}, ...,$$
$$A_{2^{i-1}} = \{2^{i-1} - 1, 2^i - 1, ..., n - 1\}.$$

In particular, $f_2$ generates 2 cycles of length $\frac{n}{2}$, one on the set of even numbered vertices, the other one on the set of odd numbered vertices. Furthermore, the partition defined by $f_i$ is a refinement of the partition defined by $f_{i-1}$ for $i \geq 2$.

More precisely, we can state the following result, that will be used in the proofs of the properties of the graphs $G_{2s}(F, \hat{Z})$ and $G_{2s+1}(F, \hat{Z}, g)$.

**Proposition 3.1.** Let $G = G_{2s}(F, \hat{Z})$ be a DCC linear congruential graph on the vertex set $\{0, 1, ..., 2^s - 1\}$, where $F = f_i: 1 \leq i \leq s$, with $f_i(x) = ax + c_i$ verifying the conditions given above.

Then, the subgraphs $G_1$ and $G_2$ induced by the vertex sets $V_1 = \{0, 2, ..., 2^s - 2\}$ and $V_2 = \{1, 2, ..., 2^s - 1\}$, are isomorphic to the DCC linear congruential graphs $G_{2s-2}(F', \hat{Z}^3)$ and $G_{2s-2}(F'', \hat{Z}^3)$ where

$$F' = \{f_i': 1 \leq i \leq s-1\},$$
$$F'' = \{f_i'': 1 \leq i \leq s-1\},$$

with

$$f_i'(x) = a_{i+1}x + \frac{c_{i+1}}{2} \quad \text{and}$$
$$f_i''(x) = a_{i+1}x + \frac{c_{i+1} + a_{i+1} - 1}{2}$$

respectively. Moreover, $G$ is the edge-disjoint union of $G_1$, $G_2$, and of the Hamiltonian cycle induced by $f_s$ which forms a bipartite graph of degree 2 on $(V_1, V_2)$.

**Proof.** For any $i$, $1 \leq i \leq s - 1$, the constant $a_{i+1}$ of functions $f_i'$ and $f_i''$ is equal to $2^{i+1}b_i + 1$ for some integer $b_i$, and the constants $c_{i+1}/2$ and $(c_{i+1} + a_{i+1} - 1)/2$ are divisible by $2^i$ but not by $2^{i+1}$ for $1 \leq i \leq s - 1$, and the graphs $G_{2s-2}(F', \hat{Z}^3)$ and $G_{2s-2}(F'', \hat{Z}^3)$ are DCC linear congruential graphs.

We will show that the functions $h_1(x) = x/2$ and $h_2(x) = (x - 1)/2$ define an isomorphism between $G_1$ and $G_{2s-2}(F', \hat{Z}^3)$, $G_2$ and $G_{2s-2}(F'', \hat{Z}^3)$, respectively. Consider first the graph $G_1$. Let $i$ be an integer, $2 \leq i \leq t$, and $x$ be a vertex in $V_1$. Since any vertex in $V_1$ is an even integer, $x = 2j$ for some $j$. Thus,

$$f_i(x) = f_i(2j) = a{j}2j + c_i = 2(a_jj + c_i / 2) = 2f_i'(j).$$

If $(x, f_i(x))$ is an edge in $G_1$, then $(h_1(x), h_1(f_i(x))) = (j, f_i(2j)/2) = (j, f_i'(j))$ is an edge in $G_{2s-2}(F', \hat{Z}^3)$.

Similarly, for every $y$ in $\{0, 1, ..., 2^{s-1}\}$, if $(y, f_i''(y))$ is an edge in $G_{2s+2}(F'', \hat{Z}^3)$ then $(h_2(y), h_2(f_i''(y))) = (2y, f_i(2y))$ is an edge in $G_1$. Thus, $h_2$ defines an isomorphism between $G_1$ and $G_{2s-2}(F'', \hat{Z}^3)$.

Consider now the graph $G_2$. Let $i$ be an integer, $2 \leq i \leq t$, and $x$ be a vertex in $V_2$. Since any vertex in $V_2$ is an odd integer, $x = 2j + 1$ for some $j$. Thus,

$$f_i(x) = f_i(2j) = a_j2j + a_i + c_i = \frac{2(a_jj + (c_i + a_i - 1)/2) + 1}{2} = \frac{2f_i''(j) + 1}{2}$$

If $(x, f_i(x))$ is an edge in $G_2$ then

$$(h_2(x), h_2(f_i(x))) = (j, f_i'(2j + 1)/2) = (j, f_i'(j))$$

is an edge in $G_{2s-2}(F', \hat{Z}^3)$. Similarly, for every $y$ in $\{0, 1, ..., 2^{s-1}\}$, if $(y, f_i''(y))$ is an edge in $G_{2s-2}(F'', \hat{Z}^3)$ then

$$(h_2(y), h_2(f_i''(y))) = (2y + 1, 2f_i''(y)/2) = (2y + 1, f_i(2y + 1))$$

is an edge in $G_2$. Thus, $h_2$ is an isomorphism between $G_2$ and $G_{2s-2}(F'', \hat{Z}^3)$.

The fact that $G$ is the edge-disjoint union of $G_1$, $G_2$, and of the Hamiltonian cycle induced by $f_s$ which forms a bipartite graph of degree 2 on $(V_1, V_2)$ is obvious.

We first consider the vertex-connectivity (connectivity for short) of the DCC linear congruential graphs.

**Theorem 3.2.** Let $t \leq p$ be an integer, and $F = f_i: 0 \leq i \leq t - 1$ be defined as above. Then the DCC linear congruential graph $G_{2s}(F, \hat{Z})$ is 2-connected.

**Proof.** The proof is by induction on $t$. The result is obviously true for $t = 1$ since in this case the DCC linear congruential graph is a cycle of length $2$ and, therefore, is 2-connected. Assume that, for any appropriate DCC set $F'$ of $t - 1$ linear functions, the graph $G_{2s-1}(F', \hat{Z}^3)$ is 2-$(t - 1)$-connected. Consider the partition of the set of vertices of a DCC linear congruential graph $G_{2s}(F, \hat{Z})$ into the vertex sets $V_1, V_2$ of even, odd numbered vertices, on which $f_t$ generates cycles of lengths $2^{i-1}$, say $C_t$ and $C_2$. By Proposition 3.1, the subgraphs $G_1$ and $G_2$ induced by $V_1$ and $V_2$ are isomorphic to DCC linear congruential graphs $G_{2s-1}(F', \hat{Z}^3)$ and $G_{2s-1}(F'', \hat{Z}^3)$, respectively which, by the induction hypothesis, are 2-$(t - 1)$-connected. Consider any two vertices $x$ and $y$ of $G_{2s}(F, \hat{Z})$.

**Case 1.** Vertices $x$ and $y$ are both in $V_1$ (see Fig. 2a).

Since by the induction hypothesis $G_1$ is 2-$(t - 1)$-connected, there exist $2t - 2$ vertex-disjoint paths between $x$ and $y$ in $G_1$. It is easy to exhibit two more vertex disjoint paths in
G_{21}(F, Z^2) depending on the relative position of the vertices \( f_i(x), f_i(y), f_i^1(x), f_i^1(y) \) on the cycle C_2 as follows. If both \( f_i^1(x) \) and \( f_i^1(y) \) are on the same part of the cycle C_2 between \( f_i(x) \) and \( f_i(y) \), then the paths are:

Path 1: \( x, f_i^1(x), \text{part of } C_2 \to f_i^1(y), y \)
Path 2: \( x, \text{part of } C_2 \text{ not containing } f_i^1(x) \to f_i(y), y \)

otherwise

Path 1: \( x, f_i(x), \text{part of } C_2 \text{ containing } f_i^1(x) \to f_i(y), y \)
Path 2: \( x, f_i(x), \text{part of } C_2 \text{ containing } f_i^1(y) \to f_i(y), y \)

Case 2. Vertex \( x \) is on \( C_1 \) and \( y \) is on \( C_2 \). Assume first that \( y \in f_i(x), f_i^1(x) \) (see Fig. 2b). Let \( S_1 \) be a subset of \( 2t - 2 \) vertices of \( V_1 - \{x, f_1^2(x)\} \) containing \( f_i(y) \) and \( f_i^1(y) \). Denote the vertices of \( S_1 \) different from \( f_i(y) \) and \( f_i^1(y) \) as \( a_1, a_2, \ldots, a_{2t-4} \). By the induction hypothesis, \( G_s \) is \( 2(t-1) \)-connected, and thus, by Menger’s theorem, there exists a set \( P \) of \( 2t - 2 \) vertex disjoint paths between \( x \) and \( y \) of the vertices of \( S_1 \) and \( y \). We are now able to exhibit \( 2t \) vertex disjoint paths between \( x \) and \( y \) in \( G_{21}(F, Z^2) \). For every \( i, 1 \leq i \leq 2t - 4 \), take the following path:

\( x, \text{path in } P_i \text{ between } x \text{ and } a_i, f_i(a_i), \text{path in } P_2 \text{ between } f_i(a_i) \text{ and } y \).

The last four paths are defined as follows.

Path 1: \( x, f_i(x), \text{path between } f_i(x) \text{ and } y \text{ in } G_{21} \)
Path 2: \( x, f_i^1(x), \text{path between } f_i^1(x) \text{ and } y \text{ in } G_{21} \)
Path 3: \( x, \text{path in } G_1 \text{ between } x \text{ and } f_i^1(y), y \)
Path 4: \( x, \text{path in } G_2 \text{ between } x \text{ and } f_i(y), y \).

The proof is very similar if \( y = f_i(x) \) (or \( y = f_i^1(x) \), respectively). In that case paths 1 and 3 (or paths 2 and 4, respectively) above are reduced to the edge \( xy \). It is possible to add one more vertex \( a_{2t-3} \) in the set \( S_1 \) so that it is still of cardinality \( 2t - 2 \) and it avoids \( f_i^1(y) \) (or \( f_i(y) \), respectively) which is equal to \( x \). Thus, following the same reasoning as above, where \( 2t - 4 \) is replaced by \( 2t - 3 \), we still have \( 2t \) vertex disjoint paths between \( x \) and \( y \).

\[ z = g_i^h(f_i(g_i^{1-h}(f_i(...(g_i^h(x)))))) \]
where \( 0 \leq j_i \leq (a - 1)/2 \) and \( g_i \) is either equal to \( f_j \) or \( f_0 \) for every \( i, 1 \leq i \leq s \). Notice that for any value of \( z \) above, there is a path from \( x \) to \( z \) mod \( Z^2 \) of length \( j_1 + j_2 + \cdots + j_s + s - 1 \) in \( G_{21}(F, Z^2) \). Since \( a \) and \( c \) are both odd, the parity of \( f_i(x) \) is different from the one of \( x \).

If \( s = 1 \) then \( z = g_i^h(x) \), and the possible values of \( z \) are \( a \) integers of the same parity as \( x \). In the set \( R_1 = \{x - (a - 1), x - (a - 1) + 2, \ldots, x + (a - 1)\} \), Consider now the possible values of \( z \) for \( s = 2 \), i.e., \( z = g_i^h(f_i(g_i^1(x))) \). First, by applying the function \( f_i \) to the elements of \( R_1 \), we obtain the set

\[ X_1 = \{f_i(x - (a - 1)), f_i(x - (a - 1) + 2), \ldots, f_i(x + (a - 1))\} \]

\[ = \{f_i(x) - (a^2 - a), f_i(x) - (a^2 - a) + 2a, \ldots, f_i(x) + (a^2 + a)\}. \]
All values in $X_i$ are of the same parity as $f_i(x)$ and the
difference between any two consecutive values in the set is
equal to $2a$. Now, by applying the function $g^1_i$ to the
values in the set $X_i$ for all values of $f_j, 0 \leq j \leq (a - 1)/2$ and $g_2$
being either $f_2$ or $f_3$, we obtain all values in the set
\[
R_2 = \{f_i(x) - (a^2 - a) - (a - 1), f_i(x) - (a^2 - a) - (a - 1) + 2, \ldots, f_i(x) + (a^2 - a) + (a - 1)\}
\]
\[
= \{f_i(x) - a^2 + 1, f_i(x) - a^2 + 3, \ldots, f_i(x) + a^2 - 1\}
\]
This set contains $a^2$ elements, all of them of the same
parity. Similarly, for any value of $s$, the possible values of
$z$ are $a^2$ integers in the set
\[
\{f^{s-1}_i(x) - (a^2 - 1), f^{s-1}_i(x) - (a^2 - 1) + 2, \ldots, (f^{s-1}_i(x) + (a^2 - 1)\}
\]
having the same parity as $f^{s-1}_i(x)$. Thus, if we choose $s$
to be the smallest integer such that $s \geq 2^{s-1}$, i.e., $s = \lceil \log_2 2^{s-1}\rceil$, then from x there is a path to any vertex of
the same parity as $f^{s-1}_i(x)$ whose length is bounded by
$s(a - 1)/2 + s - 1 = \lceil \log_2 2^{s-1}\rceil(a + 1)/2 - 1$. Since $f_1$ is a
one-to-one mapping of all the vertices of the same parity
of the graph onto the vertices of the opposite parity, by
extending all these paths by an edge defined by $f_2$ we can
reach the remaining vertices of the graph. We can con-
clude that there is a path from $x$ to any other vertex of
the graph of length not more than $\lceil \log_2 2^{s-1}\rceil(a + 1)/2$.

In Table 1, we give the values of the diameters of some
of the graphs $G_n(F, 2^p)$ and $G_{2n+1}(F, 2^p, g)$ which have a low
diameter, for $9 \leq p \leq 17$ and degrees between 3 and 10.
These values are in fact much smaller than the upper bound
from the above theorem. Each entry in the table specifies
the value of the diameter, and below the diameter we give
the constants of the linear functions generating the graph.
For graphs of order up to $2^{14}$ we have calculated the dis-
tances between all pairs of vertices of the graphs.

For graphs of larger order we have calculated the dis-
tances only for large segments of vertices.

In all the cases the DCC linear congruential graphs are
much larger than the de Bruijn graphs of the same degree
and diameter. For example, for degree 4 and diameter 10,
there is a DCC linear congruential graph of order $8,192$
while the de Bruijn graph is of order 1,024. Similarly, there
is a DCC linear congruential graph of degree 9, and diam-
eter 5 which has more vertices than the de Bruijn graph
of degree 10, and diameter 5.

The diameter of DCC linear congruential graphs is sensi-
tive to the choice of the constants of the generators. For
example, the diameter of a DCC graph increases significantly
when its set of generators contains a pair of functions
$5x + 1, 9x + 2$ which are commutative. As seen from the
tables, we obtained the best results with respect to the di-
амeter of DCC graphs when:

1) the multiplicative constants of the generators are all
distinct,

2) the multiplicative constants increase with the cycle
type of the generators, however the increases are as
small as possible,

3) the generators are not commutative, i.e., the function
$f_i + f_j$ is different from the function $f_j + f_i$ for $i \neq j$.

In general our experiments have indicated that all DCC
linear congruential graphs of same degree and order ob-
tained with the above restrictions on the generators have
almost the same diameter.

Table 2 gives additional DCC linear congruential graphs,
some of them of orders that are not powers of 2.

The results in Tables 1 and 2 lead us to propose the fol-
lowing conjecture:

**CONJECTURE 3.5.** For the functions $f_1 = 5x + 3, f_2 = 9x + 2,$

\[
diameter(G_n(f_1, 2^p, f_2)) \leq \lceil 1,2p \rceil \leq 1,2p
\]

\[
diameter(G_n(f_2, f_1, 2^p)) \leq \lceil 0,8p \rceil \leq 0,8p.
\]

Notice that the Moore bound implies that the diameter of
graphs of order $n$ and degree 3 is greater than or equal to
$\log_2 n - 2/3$, and the diameter of graphs of order $n$ and
degree 4 is greater than or equal to $0.631 \log_2 n - 1/2$. The best
general construction of graphs of order $n$ and degree 3 or 4
was given by Jerrum and Skyum [20]. For degree 3, it gives
graphs of diameter $1.47 \log_2 n + O(1)$, and for degree 4, the
diameter is $0.9083 \log_2 n + O(1)$.

In Proposition 3.1, we showed that a DCC linear congruential
graph $G_2(n, 2^p)$ of degree $2t$ and order $2^p$ can be
decomposed into two vertex disjoint DCC linear congruential
graphs $G_{2t}(n, 2^{t-1})$ and $G_{2n-2t}(n, 2^{t-1})$ of degree $2t - 2$ and
order $2^{t-1}$. Furthermore, there is a bipartite graph of degree 2
on the partition defined by the two disjoint sets of vertices.

We now give a construction of a DCC linear congruen-
tial graph of order $2^{p+1}$ from two copies of DCC linear con-
gruential graphs of order $2^p$.

**CONSTRUCTION 3.6.** A DCC linear congruential graph $G_2(n, 2^{p+1})$
can be constructed from two copies of $G_2(n, 2^p)$ as follows:

1) Denote the two copies of $G_2(n, 2^p)$ as $H_1$ and $H_2$. Renum-
ber the vertices of $H_i$ by adding $2^p$ to each vertex.

2) For every vertex $x$ of $H_1$, if $(x, y)$ is an edge in $H_1$ where
$y = f_i(x) \bmod 2^p$ and $2^p < f_i(x) \bmod 2^{p+1}$ then remove the
edge $(x, y)$ in $H_1$ and the edge $(x + 2^p, y + 2^p)$ in $H_2$ and
add instead edges $(x, y + 2^p)$ and $(x + 2^p, y)$.

Notice that in the construction above, only a fraction of the
edges is replaced since when the edge $(x, f_j(x) \bmod 2^p)$ is
changed then the edge $(2x \bmod 2^p, f_j(2x) \bmod 2^p)$ is not
changed.

Although the construction above is stated only for a DCC
linear congruential graph of even degree, it is clear that
the same construction can be also carried out for a DCC linear
congruential graph of odd degree. See in Fig. 3 a construc-
tion of $G_2(f_i, 32, g)$ from $G_2(f_i, 16, g)$ for $f_i(x) = 5x + 3, g(x) = 9x + 2$. The edges that are to be replaced are drawn in thick lines.
TABLE 1

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We would like to point out that other symmetric graphs such as hypercubes, star graphs, and pancake graphs admit a similar construction.

4 CONCLUSIONS

The DCC linear congruential graphs presented in this paper form a very interesting family of graphs. Unlike de Bruijn graphs, they are defined for both odd and even degrees, are regular and of maximum connectivity for even degrees. They can be defined for any order which contains a multiple factor. Graph \( G(F, n) \) is a proper subgraph of \( G_{\text{rat}}(F, n, f_{\text{rat}}) \) and \( G(F, 2^n) \) can be constructed from two copies of \( G(F, 2^n) \). Thus, DCC linear congruential graphs satisfy the extensibility requirements in network design. As seen from the tables, they have many more vertices than graphs of the same diameter and degree produced by any other general construction. Furthermore, a DCC linear congruential graph is Hamiltonian and can be decomposed into a very small number of edge-disjoint cycles, which reminds us of a useful property of hypercubes. Thus, the DCC linear congruential graphs should be considered as an alternative for interconnection network designs.

The problem of obtaining a better upper bound on the diameter of DCC linear congruential graphs is a very interesting open problem that requires further studies. It seems to us that any good upper bound on the diameter will require a use of number theoretical properties of the constants of the linear functions.

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REFERENCES


Jaroslav Opatrný received his first degree from Charles University, Prague, Czech Republic, in 1968, and the PhD in computer science from the University of Waterloo in 1975-1977. He was an assistant professor at the University of Alberta, Edmonton, Canada. In 1977 he joined the Computer Science Department at Concordia University, Montreal, Canada, where he is presently an associated professor. His main interests are network models, algorithms, and graph theory.

Dominique Sotteau received the Doctorat de l'Etat degree in computer science from the Université de Paris-Sud in 1980. After studying at the Ecole Normale Supérieure de Fontenay she entered the CNRS (National Research Council) in 1974. She is now a director of research in CNRS. She is leading the team of Graph Theory and Combinatorics (more than 20 researchers) in the Laboratoire de Recherche en Informatique of the Université de Paris-Sud in Orsay. Her research group is involved in classical graph theory, algorithms, combinatorial optimization, interconnection networks and parallel and distributed algorithms. Dr. Sotteau has published more than 45 papers, all in international well-known journals or conference proceedings, in the field of graph theory and, recently, mainly on interconnection networks. She has organized several conferences or workshops on the subject and is involved in the scientific committees of others.

N. Srinivasan received the MSc degree from Vivekananda College, University of Madras, India, in 1963, the M.Phil degree from Ramanujan Institute of Advanced Mathematics, Madras University in 1977, and the doctorate degree from the Indian Institute of Technology, Madras in 1983. He was a postdoctoral fellow at Concordia University, Montreal, Canada from 1984 to 1985. He is presently a professor of mathematics in the A.M. Jain College, Madras University. His main interests include graph theory, combinatorial optimization, Hamiltonian cycles, and fault-tolerant network models.

K. Thulasiraman (M72-SM84-F90) received the Bachelor’s and Master’s degrees in electrical engineering from the University of Madras, in 1968. He joined the Department of Electrical Engineering, Indian Institute of Technology, Madras, in 1968, and the Dept. of Computer Science in 1973. He was promoted to professor in January 1977. After serving for a year (1981-1982) in the Dept. of Electrical Engineering, Technical University of Nova Scotia, Halifax, N.S., Canada, he joined Concordia University, Montreal, P.Q., Canada, as a professor in the Dept. of Mechanical Engineering. In 1984 he moved to the Dept. of Electrical and Computer Engineering at Concordia University and is currently a regional editor of the Journal of Circuits, Systems and Computers. He was technical program chair of the IEEE International Symposium on Circuits and Systems in 1993. His current research interests are in graph theory, parallel and distributed computing, operations research and computational graph theory with applications in VLSI design automation, computer networks, communication network planning, etc.