Performance of Buffered Multistage Interconnection Networks in Non Uniform Traffic Environment

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Abstract

Multistage interconnection networks (MIN) are used to connect processors to memories in shared memory multiprocessor systems. A generalized Markov chain model for the performance evaluation of a single-buffered Omega network, in the presence of a hot spot, has been proposed in this paper. The proposed model produces better results than existing models.

1 Introduction

Multistage interconnection networks (MIN) are used to interconnect a large number of processors to memories in tightly coupled multiprocessor systems. MINs have also been proposed as switching fabrics in ATM switches of Broadband ISDN networks. The performance of unbuffered and buffered MINs have been widely studied using analytical models and simulation techniques [1,2,3,4,5]. Most of the analytical models have assumed uniform traffic in the network. Jenq [1] proposed an elegant iterative Markov chain solution, for a MIN using 2×2 switching elements (SEs). His work was generalized to $a \times a$ SEs by Yoon [2]. Theimer [3] removed two independence assumptions of Jenq to obtain better results. Hsiao [4] proposed a model by relaxing an independence assumption of Yoon. Pfister [6] reported the phenomenon of tree saturation due to hot spots in a buffered MIN. Hot spots give rise to non-uniform traffic in the network. No efficient analytical model is available to study the performance of buffered MINs in the presence of a hot spot. The aim of this paper is to develop an analytical model for the performance evaluation of single-buffered MINs in the presence of a hot spot.

Jenq [1] laid the groundwork for analyzing a single-buffered Omega network having 2×2 SEs. He made the following *independence assumptions* in order to reduce the state space of the model.

1. The state of a SE at stage k is statistically not distinguishable from that of another SE at the same stage.
2. Routing requests from packets in the buffer of a SE at successive stage cycles are independent.
3. The two buffers in the same SE are statistically independent.

Based on the above assumptions, Jenq proposed a 2-state model where the network was modeled by the state of a single buffer being empty or full. A packet, blocked during a cycle for a particular output link of a SE, will request the same output link of the SE at all successive cycles until it moves to the next stage. Therefore, assumption 2 of Jenq's model does not hold for blocked packets. Secondly, the probability that a packet in a buffer of a SE can move forward during a cycle *depends* on the state of the other buffer of the SE during that cycle. The above fact is in contradiction to assumption 3 of Jenq's model.

Jenq proposed a second model by taking into account the dependencies between the buffers of a SE. He concluded that *the assumption of independence between buffers is a good approximation.*

Jenq's work has been extended by Yoon [2] to model a multiple-buffered Delta network using $a \times a$ SEs. Theimer [3] developed a model by removing independence assumptions 2 and 3 of Jenq. Theimer concluded that *consideration of the dependencies between buffers of a SE leads to a small improvement of the results.* Hsiao [4] proposed an extension to Yoon's model by taking into account the dependency between states of the buffers of a SE, and the fact that a blocked packet will hunt for a previously determined output link. The inaccuracy in the model is due to its inability to store the information regarding the output link of the SE for which the packet was blocked.

We modified the above mentioned existing models for uniform traffic, in the hope of arriving at a model which would be suitable for hot spot traffic. The modifications included the following changes to the previous assumptions.

- □ The routing probabilities of a new packet to the output links of a hot SE, are proportional to the sum of the probabilities of the requests to the set of memories that can be reached through the output links.
- □ Since hot spot traffic causes different traffic rates at the inputs of the SEs of a stage, the SEs of a stage are not statistically identical. Classified according

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to the data rates at the inputs to the switches [7], S_i has i types of SEs.

We propose a 4-state memorized model which stores the information regarding the output link that was requested when a packet was blocked. In addition to removing both the independence assumptions of Jenq and Yoon, we have taken a rigorous account of the dependencies among successive requests from a buffer in a SE. We have further extended the models proposed by Jenq and Theimer to make them suitable for hot spot traffic by having different sets of equations for the different types of SEs. In a similar manner, we have extended our proposed model to permit the analysis in the presence of hot spot traffic.

Because of space limitations, our proposed model will be described only for uniform traffic. Extensions to the model for hot spot traffic can be easily done by using the properties of a MIN under hot spots [7]. However, *results obtained from Jenq's model, Theimer's model, and our proposed model will be presented for both uniform and hot spot traffic patterns.*

The objectives of the research work described in this paper are as follows.

- □ To modify the existing models to make them suitable for hot spot traffic, and evaluate their accuracy.
- □ To develop a new generalized analytical model for hot spot traffic. Uniform traffic can also be analyzed by the generalized model since the uniform traffic is a special case of the hot spot traffic.

The modeling assumptions are described in the next section. A generalized model is proposed in section 3. Results obtained from the proposed model, in the presence of uniform and hot spot traffic, are compared with those from previous models and stochastic simulations in section 4. Finally, some concluding remarks are given in section 5.

2 Modeling Assumptions

The Omega network will be taken as an example of a MIN to be modeled. The network connects N inputs to N outputs using $n = \log_2 N$ stages of $N/2$ SEs per stage. The following modeling assumptions, used by most authors, are made regarding the network and its operation.

1. There are $N = 2^n$ processors and N memory modules in the system, where n is an integer. The i−th processor and the j−th memory will be denoted by PE_i and MM_j respectively, where $0 \leq i, j \leq N-1$.
2. A *packet switched* network operating in the *synchronous mode* is assumed. *Destination tag* routing is used to route packets in the network.
3. Each input to a SE has a single buffer to store a packet. If two packets at the input buffers of a SE request the same output link, the SE *randomly* selects one input; the rejected one is blocked in the buffer and has to try again in the next cycle.
4. *Temporal independence* of requests is assumed.
5. Memory requests from processors, during a cycle, are assumed to be *spatially independent.*
6. The probability that a processor generates a memory request at the beginning of a cycle is p_0.
7. For uniform traffic, the probability that a processor request is directed to any particular memory module is $1/N$. For hot spot traffic pattern, MM_h is assumed to be a *hot memory module* for all processors. If PE_i generates a request during a cycle, the probability that it will request MM_h is h, whereas the probability of requesting any other module MM_j, $j \neq h$, is $h' = (1-h)/(M-1)$, where $h > h'$.
8. In the case of uniform traffic, a new packet arriving at the input of a SE is equally likely to request any one of the output links of the SE. On the contrary, for hot spot traffic pattern, a packet arriving at the input of a SE is not necessarily uniformly distributed over the outputs of the SE.
9. A *backpressure mechanism* prevents any loss of packet inside the network.
10. There is *no blocking* at the *output links* of the network.
11. Requsts for hot memory modules are called *hot requests.* Links and switches which carry hot requests are called *hot links* and *hot switches* respectively.

Assumption 4 is unrealistic since, in practice, blocked requests are resubmitted in the next cycle. The assumption makes the analysis simple without introducing too much of errors [8].

3 Performance Analysis

The performance measure considered is the average bandwidth of the network. The average bandwidth is defined as the average number of packets arriving at the output of the network during a stage cycle. A Markov chain model has been developed by assuming that the SEs in a stage are statistically independent in the case of uniform traffic, and then assigning states to the buffers of a SE. The 4-state model memorizes the history of blocking. The states of a buffer in a SE are represented by $0, n, b_u$ and b_l corresponding to the buffer being in an empty state, new state, blocked for the upper output link state, and blocked for the lower output link state respectively. For modeling purposes, we split the stage cycle time (τ) into two phases as in [1,2,3].

- □ In the *first phase* of the cycle (τ_1), the availability of buffer space at the next stage along the destined path of a packet in the current buffer is determined.
- □ In the *second phase* (τ_2), packets may move forward one stage if the next stage buffers are ready to accept them.

The whole process is repeated every stage cycle. However, in practice the two phases in a buffer overlap, and enqueuing and dequeuing of packets in a buffer occur simultaneously. We call the states of the buffer at the beginning of τ_1, at the end of τ_1, and at the end of τ_2 as *initial* state, *intermediate* state, and *final* state respectively.

The initial (p_x) and intermediate ($\tilde{p}_x$) state probabilities of a buffer at stage k during the t-th stage cycle are defined as follows.

$p_x(t,k)$ = Pr[a buffer at stage k is in state x at the beginning of τ_1 of the t-th stage cycle, $x \in \{0, n, b_u, b_l\}$].

$\tilde{p}_x(t,k)$ = Pr[a buffer at stage k is in state x at the end of τ_1 of the t-th stage cycle, $x \in \{0, b_u, b_l\}$].

The routing probability of a packet is defined as the probability that a packet in a buffer of a SE can be routed to the next stage. The routing and blocking probabilities will be denoted by r and $\bar{r}$ respectively and are defined as follows.

$r_x^u(t,k)$ = Pr[a packet in a buffer of a SE at stage k is *able* to move forward to the *upper* output link of the SE during τ_1 of the t-th stage cycle, given that the buffer was in state x at the beginning of the cycle, $x \in \{0, n, b_u, b_l\}$].

$\bar{r}_x^u(t,k)$ = Pr[a packet in a buffer of a SE at stage k is *unable* to move forward to the *upper* output link of the SE during τ_1 of the t-th stage cycle, given that the buffer was in state x at the beginning of the cycle, $x \in \{0, n, b_u, b_l\}$].

Similarly, $r_x^l(t,k)$ and $\bar{r}_x^l(t,k)$ are the routing and blocking probabilities respectively for the lower output link of a SE. The acceptance probabilities (α) of a buffer are defined as follows.

$\alpha(t,k)$ = Pr[a buffer in a SE at stage k is *able to accept* a packet during τ_2 of the t-th stage cycle].

$\alpha_x(t,k)$ = Pr[a buffer in a SE at stage k is *able to accept* a packet during τ_2 of the t-th stage cycle, given that the buffer was in state x at the beginning of the stage cycle].

The traffic rate at a link between two stages depends on whether a packet is offered to the next stage through the link and the next stage is able to accept it. The probability of offering a packet to the next stage and the traffic rate at a link will be denoted by q and ρ respectively.

$q(t,k)$ = Pr[a packet is *offered* to a buffer of a SE at stage k during τ_2 of the t-th stage cycle].

The offered packet is received by the buffer at stage k if the buffer was in state 0 at the start of τ_1, or in any of the states n, b_u, or b_l at the start of τ_1 and the packet was successfully routed to stage $k+1$ during τ_1.

$\rho^u(t,k)$ = Pr[a packet is received on the upper input buffer of a SE at stage k during τ_2 of the t-th stage cycle].

Similarly, $\rho^l(t,k)$ is the probability that a packet is received on the lower input buffer of a SE. The following two lemmas will be used in deriving the expressions for the different state and transition probabilities.

Lemma 1: When the two buffers of a SE at stage k contain a new and a blocked packet at the start of the t-th stage cycle, then the buffer at stage $k+1$ for which the blocked packet (at stage k) is destined can not be in state 0.

Lemma 2: At the final stage of the network, both the buffers of a SE can not be in the same (either b_u or b_l) blocked state during any cycle.

Figure 2 shows the possible state transitions from the initial states of a buffer (beginning of τ_1) to intermediate states (end of τ_1), and from intermediate states to final states (end of τ_2). The probabilities of the different state transitions are also shown in the figure. The intermediate and initial state probabilities can be related as follows.

Intermediate state probabilities

$$\tilde{p}_0(t,k) = p_0(t,k) + p_n(t,k)[r_n^u(t,k) + r_n^l(t,k)] + p_{b_u}(t,k)r_{b_u}^u(t,k) + p_{b_l}(t,k)r_{b_l}^l(t,k) \quad (1)$$

$$\tilde{p}_{b_u}(t,k) = p_n(t,k)\bar{r}_n^u(t,k) + p_{b_u}(t,k)\bar{r}_{b_u}^u(t,k) \quad (2)$$

$$\tilde{p}_{b_l}(t,k) = p_n(t,k)\bar{r}_n^l(t,k) + p_{b_l}(t,k)\bar{r}_{b_l}^l(t,k) \quad (3)$$

State probabilities

$$p_0(t+1,k) = \tilde{p}_0(t,k)[1-q(t,k)] \quad (4)$$

$$p_n(t+1,k) = \tilde{p}_0(t,k)q(t,k) \quad (5)$$

$$p_{b_u}(t+1,k) = \tilde{p}_{b_u}(t,k) \quad (6)$$

$$p_{b_l}(t+1,k) = \tilde{p}_{b_l}(t,k) \quad (7)$$

Acceptance probabilities

The probability that a buffer at stage k can accept a packet during the t-th cycle can be expressed in terms of the routing and intermediate state probabilities as given below.

$$\alpha_n(t,k) = r_n^u(t,k) + r_n^l(t,k) \quad (8)$$

$$\alpha_{b_u}(t,k) = r_{b_u}^u(t,k) \quad (9)$$

$$\alpha_{b_l}(t,k) = r_{b_l}^l(t,k) \quad (10)$$

$$\alpha(t,k) = \tilde{p}_0(t,k) \quad (11)$$

Routing and blocking probabilities

When there is a *new* packet and a *blocked* packet at the two buffers of a SE at stage k during cycle t, and both of them are destined for the same output link of the SE, the probability that the buffer at stage $k+1$ is able to accept one of these packets is found as follows. The probability that the buffer at stage $k+1$ is in state n at time t is equal to the probability that it received a packet during cycle $t-1$ from the buffer at stage k which contains the new packet at cycle t. This probability is given by $0.5\rho(t-1,k+1)$. Since the next stage can not be in an empty state under the above circumstances, the probability that the next stage is in a blocked state (either b_u or b_l) is $1-0.5\rho(t-1,k+1)$. For uniform traffic in the network, the probability that it is in state b_u or b_l is equal. Thus the probability that is in state b_u (or b_l) is $[1-0.5\rho(t-1,k+1)]/2$. In such a case, let's denote the probability that the buffer at stage $k+1$ can accept a packet by α_1 which is given by

$$\alpha_1(t,k+1) = 0.5\rho(t-1,k+1)\alpha_n(t,k+1) + 0.5[1-0.5\rho(t-1,k+1)][\alpha_{b_u}(t,k+1) + \alpha_{b_l}(t,k+1)] \quad (12)$$

When *both the buffers* in a SE at stage k during cycle t are in *blocked* states, the next stage buffer must be in a blocked state. The probability that the buffer at stage $k+1$ can accept a packet is, therefore, given by

$$\alpha_2(t,k+1) = 0.5[1-0.5\rho(t-1,k+1)][\alpha_{b_u}(t,k+1) + \alpha_{b_l}(t,k+1)] \quad (13)$$

With the above probabilities of acceptance, we can express the routing probabilities for the packets at stage k, $1 \le j \le n-1$, during cycle t as follows.

$$\begin{aligned} r_n^u(t,k) &= 0.25p_n(t,k)\alpha(t,k+1) + 0.125p_n(t,k)\alpha(t,k+1) + 0.5p_0(t,k)\alpha(t,k+1) + 0.25p_{b_u}(t,k)\alpha_1(t,k+1) + 0.5p_{b_l}(t,k)\alpha(t,k+1) \\ &= [0.375p_n(t,k) + 0.5p_0(t,k) + 0.5p_{b_l}(t,k)]\alpha(t,k+1) + 0.25p_{b_u}(t,k)\alpha_1(t,k+1) \quad (14) \end{aligned}$$

$$r_n^l(t,k) = [0.375p_n(t,k) + 0.5p_0(t,k) + 0.5p_{b_u}(t,k)]\alpha(t,k+1) + 0.25p_{b_l}(t,k)\alpha_1(t,k+1) \quad (15)$$

Given that a buffer is in a new state, the sum of the routing and blocking probabilities of a packet must be equal to one. Therefore,

$$r_n^u(t,k) + r_n^l(t,k) + \bar{r}_n^u(t,k) + \bar{r}_n^l(t,k) = 1 \quad (16)$$

Moreover, for uniform traffic in the network the routing and blocking probabilities of a new packet for the upper or lower output links are the same, i.e.,

$$r_n^u(t,k) = r_n^l(t,k) \quad (17)$$

$$\bar{r}_n^u(t,k) = \bar{r}_n^l(t,k) \quad (18)$$

Substituting equations (17) and (18) in (16), we get

$$\bar{r}_n^u(t,k) = 0.5 - r_n^u(t,k) \quad (19)$$

$$\bar{r}_n^l(t,k) = 0.5 - r_n^l(t,k) \quad (20)$$

Given that a packet at stage k, $1 \le k \le n-1$, during cycle t is in a blocked state, the routing and blocking probabilities for the packet can be expressed as follows.

$$\begin{aligned} r_{b_u}^u(t,k) &= 0.5p_n(t,k)\alpha_1(t,k+1) + 0.25p_n(t,k)\alpha_1(t,k+1) + p_0(t,k)\alpha_1(t,k+1) + 0.5p_{b_u}(t,k)\alpha_2(t,k+1) + p_{b_l}(t,k)\alpha_2(t,k+1) \\ &= [0.75p_n(t,k) + p_0(t,k)]\alpha_1(t,k+1) + [0.5p_{b_u}(t,k) + p_{b_l}(t,k)]\alpha_2(t,k+1) \quad (21) \end{aligned}$$

$$r_{b_l}^l(t,k) = [0.75p_n(t,k) + p_0(t,k)]\alpha_1(t,k+1) + [0.5p_{b_l}(t,k) + p_{b_u}(t,k)]\alpha_2(t,k+1) \quad (22)$$

$$\bar{r}_{b_u}^u(t,k) = 1 - r_{b_u}^u(t,k) \quad (23)$$

$$\bar{r}_{b_l}^l(t,k) = 1 - r_{b_l}^l(t,k) \quad (24)$$

Equations (23) and (24) follow from the fact that a packet in a blocked state for a particular output link can either be routed to that particular output link or has to remain in the same blocked state.

It has been assumed in section 2 that there is no blocking at the last stage of the network. The routing and blocking probabilities for the last stage ($k=n$) can be obtained from the equations for stage k, $1 \le k \le n-1$, by replacing the next stage acceptance probabilities by unity. The resulting equations for $k=n$ are as follows.

$$\begin{aligned} r_n^u(t,n) &= 0.25p_n(t,n) + 0.125p_n(t,n) + 0.5p_0(t,n) + 0.25p_{b_u}(t,n) + 0.5p_{b_l}(t,n) \\ &= 0.5 - 0.125p_n(t,n) - 0.25p_{b_u}(t,n) \quad (25) \end{aligned}$$

$$r_n^l(t,n) = 0.5 - 0.125p_n(t,n) - 0.25p_{b_l}(t,n) \quad (26)$$

$$\bar{r}_n^u(t,n) = 0.5 - r_n^u(t,n) \quad (27)$$

$$\bar{r}_n^l(t,n) = 0.5 - r_n^l(t,n) \quad (28)$$

$$r_{b_u}^u(t,n) = 1 - 0.25p_n(t,n) \quad (29)$$

$$r_{b_l}^l(t,n) = 1 - 0.25p_n(t,n) \quad (30)$$

$$\bar{r}_{b_u}^u(t,n) = 1 - r_{b_u}^u(t,n) \quad (31)$$

$$\bar{r}_{b_l}^l(t,n) = 1 - r_{b_l}^l(t,n) \quad (32)$$

<u>Traffic rates at the links</u>

The traffic rates will be the same for all output links at the last stage in the case of uniform traffic, but will be different in the presence of hot spot traffic. The traffic rate at any link connecting the SEs between stages $k-1$ and k (or the load applied to any stage k) can be obtained by finding the probability that a packet is received in a buffer of a SE at stage k from the SE at stage $k-1$. The traffic rates at an upper or lower input link of a SE at stage k are given by equations (33) and (34) respectively.

$$\rho^u(t,k) = 2[p_n(t,k-1)r_n^u(t,k-1) + p_{b_u}(t,k-1)r_{b_u}^u(t,k-1)] \quad (33)$$

$$\rho^l(t,k) = 2[p_n(t,k-1)r_n^l(t,k-1) + p_{b_l}(t,k-1)r_{b_l}^l(t,k-1)] \quad (34)$$

For uniform traffic in the network, $\rho^u(t,k) = \rho^l(t,k) = \rho(t,k)$. $\rho^u(t,k)$ and $\rho^l(t,k)$ are different for hot spot traffic in the network.

The probability that a packet is offered to a buffer of a SE at stage k is the ratio of the probability that a packet is received by the buffer to the probability that the buffer can accept the packet.

$$q(t,k) = \rho(t,k)/\alpha(t,k) \tag{35}$$

The iterative method described in [1] have been used to solve the above set of equations. The average bandwidth of the network is obtained by summing the data rates at all the output links of the network. For uniform traffic, the bandwidth is given by $N\rho(n+1)$, where $\rho(n+1)$ is the steady state value of equation (33) for $k = n+1$.

4 Results

To validate our proposed analytical model, stochastic simulation was carried out both for uniform and hot spot traffic. Results obtained from the different models for uniform and hot spot traffic in an 8×8 Omega network are given below.

A comparison of the results from Jenq's model, Theimer's models, the proposed model, and simulation are given in figure 3. Since Jenq's model does not account for blocked packets, the errors increase with an increase in blocking at the SEs due to a high network traffic rate. Since blocked packets are taken into account partially in the Theimer's model, the results are closer to simulation than those from Jenq's model. It is observed that Theimer's model and our proposed model produce similar results for uniform traffic. This is because the *inaccuracy in Theimer's model does not show up until there is significant blocking in the network* as illustrated below.

Uniform traffic causes less blocking in the SEs than caused by hot spot traffic. Consequently, the accuracy of Jenq's and Theimer's models are better for uniform traffic than for hot spot traffic, as is evident from figure 4 which is a comparison of bandwidths obtained from simulation, Jenq's model, Theimer's model and the proposed model under hot spot traffic pattern for $N = 8$ and $h = 0.3$. Since Theimer's model does not completely account for a blocked packet, the errors have been found to rise with increased blocking in the network due to hot spot traffic. The proposed model produces results which are close to simulation, and are *significantly better than those obtained from modified Jenq's or Theimer's models*. The reason for improved results from the proposed model is its ability to memorizes the history of a blocked packet, whereas Theimer's model does not memorize the history and Jenq's model does not even consider blocked packets.

The effect of taking a rigorous account of blocking in our model is reflected in the increased accuracy in results in the presence of significant blocking in the network. In the presence of hot spot traffic, Theimer's model performs as well as the proposed model only when the blocking in the network is insignificant. As an example, Theimer's model produces good results for $0 \leq q(t,1) \leq 0.2$ and $h = 0.3$ (see figure 4) because of insignificant blocking for this range of network traffic rate. However, if the hot spot probability increases, Theimer's model produces large errors even for the above range of traffic rate.

5 Conclusions

Previous analytical models have, in most cases, assumed uniform traffic in the network. A new generalized analytical model for a single buffered Omega network, based on the Markov chain, has been presented. The model allows the investigation of performance of MINs in the presence of hot spot or uniform traffic. In the presence of hot spot traffic, the proposed model produces significantly better results than those by the modified versions of previous models. For uniform traffic (which is a special case of hot spot traffic), the results obtained from the proposed model are as good as results obtained from the best previous model. The inaccuracy of previous models becomes magnified in the presence of considerable blocking in the network. The proposed model can be extended to other types of non-uniform traffic patterns and to Banyan type networks.

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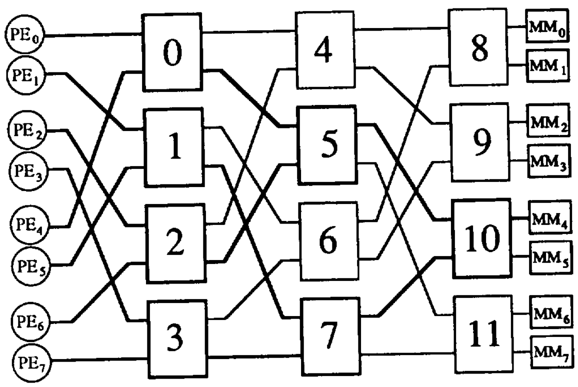

Figure 1: An 8 × 8 Omega network.

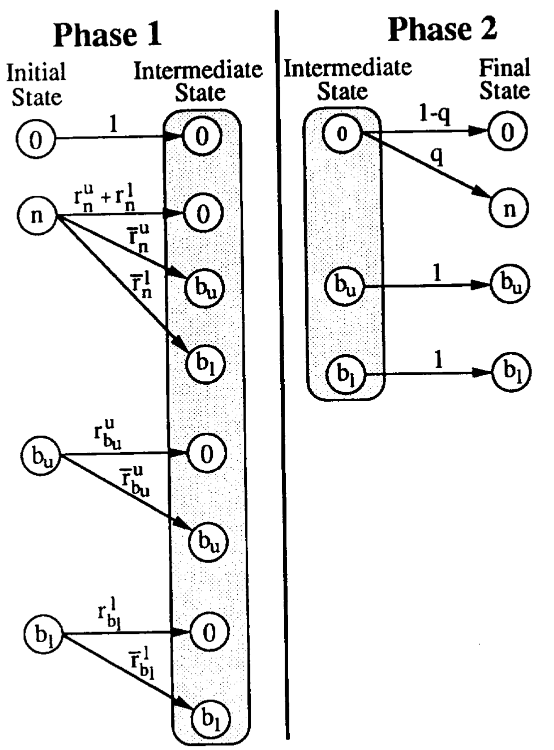

Figure 2: State transitions diagram of a buffer at a switching element.

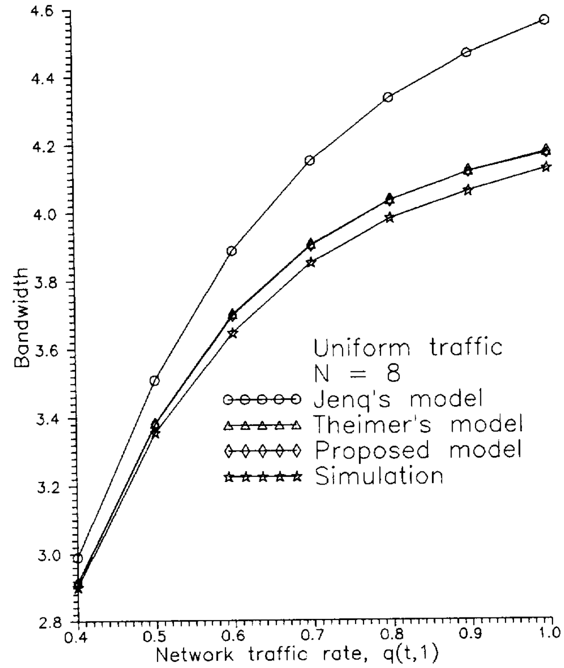

Figure 3: Comparison of bandwidths obtained from Jenq's, Theimer's, the proposed model, and simulation in the presence of uniform traffic.

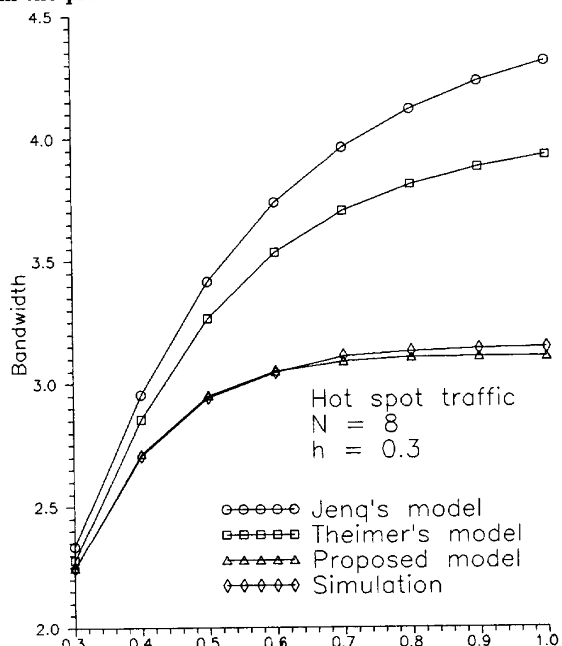

Figure 4: Comparison of bandwidths obtained from Jenq's, Theimer's, the proposed model, and simulation for $h = 0.3$.