Possibilistic evidential reasoning systems on systolic arrays

S.M. Mohiddin., M. Atiquzzaman and T.S. Dillon
Dept. of Computer Science and Computer Engineering
and Applied Computing Research Institute
La Trobe University, Melbourne, VIC 3083, Australia
email: {sayyad, atiq, tharam}@lat.oz.au

Abstract: This paper suggests a systolic array implementation of fuzzy expert systems based on possibilistic evidential reasoning methodology. Evidential reasoning systems are compute-bound in general and fuzzy systems match larger number of rules than in a simple symbolic reasoning system. The possibilistic evidential system proposed which is a combination of both needs much more computation than either of these two independently. To speed up processing in such systems the suggested implementation is useful.

1. Introduction.

Evidential reasoning systems will have a set of decisions as consequents as opposed to a single decision in rule-based expert systems. Given a hypothesis and a set of facts, the evidential reasoning systems try to assign upper and lower bounds of credibility to the hypothesis induced by the given facts through the rules in the system. In [10] this methodology has been extended to fuzzy sets by allowing the facts, rules and hypothesis to be fuzzy sets and a dual possibilistic evidential measures to assign possibility bounds to hypothesis induced by fuzzy facts through fuzzy rules. But this method takes higher computation time due to long fuzzy processing functions and computations for partially matched rules.

Recently several ways to speed up inferencing in reasoning systems has been suggested [2-5]. Some of these works are specifically meant for simple fuzzy implication. In Togai et.al., [5] and Manzoul et. al.,[2] they have used simple min-relational composition and max-min inference. Systolic architectures of [3] use simple And & Or operations on certainty factors. Other hardware architectures [4, 6] are specific to given applications. In this paper a general purpose systolic array architecture for possibilistic evidential reasoning has been proposed. The implementation provided here uses fuzzy implication as an instruction. This can be changed as necessary and deals with more complex possibilistic evidential reasoning systems. Evidential reasoning systems are compute-bound in general [12] and fuzzy systems match larger number of rules than in a simple symbolic reasoning system. The possibilistic evidential system proposed which is a combination of both needs much more computation than either of these two independently.

Possibilistic evidential reasoning approach used here is a backward chaining method, which enables all the rules whose consequents have a match or partial match with the given hypothesis fuzzy set based on a similarity measure [11]. Once chains are established from the hypothesis to the facts through the rules, fuzzy inferencing operations will compute the possibility distributions induced by the facts on to the hypothesis space. The terminology and reasoning model used are summarised in the following section. The forward computation of possibility distributions is implemented using systolic arrays in this paper.

2. Possibilistic evidential reasoning model

The reasoning model can be visualised as a disc of rings with a target disc at the centre. Each ring of the disc is made up of several frames of mutually exclusive and exhaustive set of elements. These frames are the frames of discernment in the evidential reasoning

0-7803-1896-X/94 $4.00 ©1994 IEEE

model. The outermost ring is the evidence space which absorbs the facts from the outer environment. The target disc contains frames on which the reasoning model draws its inference. Let its frames be called target frames. The directed arcs (edges) start from a frame in an outer ring and connect to a frame in an inner ring, which corresponds to the rules of reasoning. Figure 3. shows the evidential reasoning model for the fuzzy washing machine evidential reasoning model as an example.

Let $S=\{E_1, E_2, \ldots\}$ be an evidence space, a set of frames for facts and some rule antecedents, corresponding to the evidence ring of the possibilistic evidential reasoning model. Let $T=\{T_1, T_2, \ldots\}$ be a reasoning space, a set of frames for hypotheses and some rule consequents, corresponding to the target disc of the evidential reasoning model. Let $I_j=\{I_{j1}, I_{j2}, \ldots\}$ be a set of frames corresponding to the j-th inner ring in the reasoning model. Let $\Phi(X)$ be a set of all fuzzy sets on a frame X.

A reasoning chain is an element of the relation $\mathcal{R}$ defined as

$$\mathcal{R} = \{ (\Phi(E_i)\times\Phi(I_{jp})) \times (\Phi(I_{jp})\times\Phi(I_{kq})) \ldots (\Phi(I_{lr})\times\Phi(I_{ms})) \times (\Phi(I_{ms})\times\Phi(T_n)) \} \quad (1)$$

The joint possibility distribution Δ of a reasoning chain, ((A,B), (C,D), . . .,(O,P), (Q,R)), where $A\in\Phi(E_i)$, $B,C\in\Phi(I_{jp})$, . . ., $P,Q\in\Phi(I_{ms})$ and $R\in\Phi(T_n)$, is defined as follows:

$$\Delta_{R/A} = (\Pi_{B/A}, \Pi_{D/C}, \ldots, \Pi_{P/O}, \Pi_{R/Q}) \quad (2)$$

Now given the possibility distribution on the frames of the evidence space a set of basic possibility distributions (*bpds*) $\{\gamma_{ij} \mid \gamma_{ij}\in\Phi(T_i)\}$ can be induced on the target disc T_i through the reasoning chains as a fuzzy composition of the evidence (facts) possibility distribution and Δ of the reasoning chain as follows:

$$\gamma_{ij} = \Pi_E \circ \Delta_{M/N} = ((((\Pi_E \circ \Pi_1) \circ \Pi_2) \circ \Pi_3) \ldots) \quad (3)$$

where $M\in\Phi(T_i)$, $E,N\in\Phi(E_i)$, $\circ$ is Sup-Min composition [8] and $(\Pi_1, \Pi_2, \Pi_3, \ldots) = \Delta_{M/N}$ as defined in (2).

In line with the notation of evidential reasoning all non-empty γ_{ij}'s are fuzzy focal elements of frame T_i. Now the credibility bounds of any fuzzy hypothesis $H\in\Phi(T_i)$ can be obtained from the formulae of fuzzy belief (fb) (a lower bound) and fuzzy plausibility (fp) (an upper bound) [10] as follows:

$$\mathrm{fb}(H) = \operatorname*{Max}_{D} \{\mathrm{Nec}(H ; D)\} \qquad \text{for all } \mathrm{core}(D) \subseteq H \quad (4)$$

$$\mathrm{fp}(H) = \operatorname*{Max}_{D} \{\mathrm{Poss}(H ; D)\} \qquad \text{for all } \mathrm{core}(D)\cap H \neq \varnothing \quad (5)$$

Where Ds are fuzzy focals of frame T_i, $\cap$ and $\subseteq$ are fuzzy set intersection and subset operations respectively, core(X) = { x | x is a full member of X } and

$$\mathrm{Poss}(X;Y) = \operatorname*{Sup}_{\Omega} \{ \mathrm{Min}(X(x), Y(x)) \} \qquad \text{for all } x\in\Omega \quad (6)$$

$$\mathrm{Nec}(X;Y) = \operatorname*{Inf}_{\Omega} \{ \mathrm{Max}(X(x), \neg Y(x)) \} \qquad \text{for all } x\in\Omega \quad (7)$$

where Ω is reference set of fuzzy sets X and Y and $\neg$ is fuzzy complement operation.

3. Systolic array architecture for the possibilistic evidential reasoning (PER) model

Different types of Multiprocessor architectures like multiple-bus, ring, hypercube, etc., could be used to implement the PER system described in Section 2. Multiple-bus, ring and hypercube are ideal for implementation of general purpose algorithms. Systolic arrays are suitable for implementing dedicated algorithms having a regular structure.

The systolic architecture was first proposed by Kung [1] in 1982. The architecture is very suitable for a large class of regular and symmetric algorithms, like matrix multiplication. A systolic architecture consists of a large number of processing elements connected together using an interconnection network which reflects the flow of data among the processing elements. The crux of the architecture is that once a data item has been retrieved from memory, it should be used by all the processing elements which require it. This helps alleviate the processor-memory communication bandwidth problem experienced even by the fastest Von-Neumann machine. Systolic architectures reduce the complexity of the algorithms by exploiting the regularity in the algorithm. They can therefore be applied to speed up the processing in PER systems.

Implementing possibilistic evidential reasoning essentially needs implementing Equations (3) to (7). Equation (3) is implemented by reasoning process modules (see figure 3). It has the evidence $\{E_i, E_j, \ldots E_k\}$ from the set of evidences and the matching reasoning chain $\{R_x, R_y, \ldots R_z\}$ from the set of chains as inputs. From the inputs it computes the *bpds* (γ), $\{\Pi_{E_i} \circ \Pi_{R_x}, \Pi_{E_j} \circ \Pi_{R_y}, \ldots, \Pi_{E_k} \circ \Pi_{R_z}\}$, as a composition of evidence possibility distribution and joint possibility distribution of the reasoning chains. The systolic array implementation for reasoning process module is shown in Figure 1.

For illustration, we have shown the details of one reasoning process module having possibility distributions of evidence and hypothesis (target) sets defined on a reference set of cardinality three. Each reasoning process consists of three systolic arrays each of which produces an element of *bpds* (γ). Equations (6) and (7)are implemented in the P and N modules respectively as shown in Figure 2. The function carried out by the P (possibility) and N (necessity) modules are the same as those of the reasoning process module. Hence the structure of P and N modules is same as that in figure 1. The instructions executed in different modules of Figure 2 are shown in Table 1. Each PE inside a P module has two

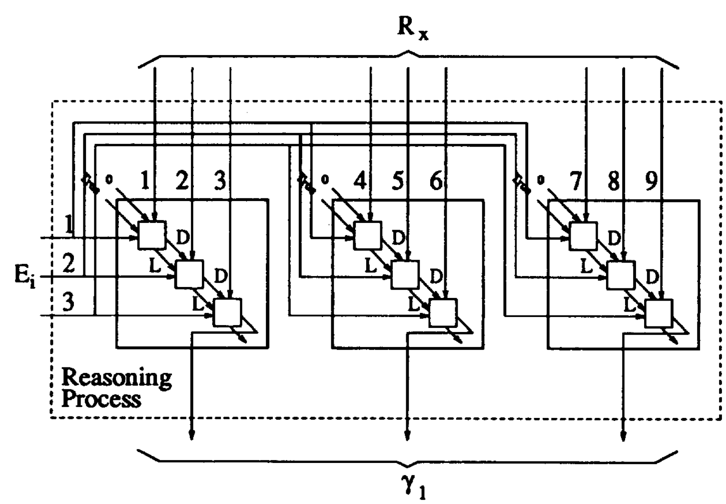

Figure 1. Systolic array implementation of reasoning process

inputs from the preceding PE. It also has an element of the possibility distributions of the hypothesis (H) and *bpds* (γ) as inputs. Each PE computes the minimum of the H and γ elements at its inputs and then finds the maximum of this minimum and the data (D) coming from the preceding PE. The values in the logic channel (L) propagates conditions required in Equations (4) and (5). The outputs of the data channel (D) of the P and N modules contain the results of Equations (6) and (7) respectively, if the conditions in (4) and (5) are satisfied, otherwise the results are 0.

The output of P and N are fed to two maximum finding modules to compute the Plausibility and Belief. The systolic array implementation of the maximum finding modules is widely available in the literature. From the *bpds*, (γ) and the given hypothesis H_q, systolic arrays $P_1,..P_m$ and $N_1,..N_m$ compute the Possibility and Necessity from equations (4) and (5). The outputs of the systolic arrays are fed to two different maximum finding units to compute the Plausibility and Belief as shown in (6) and (7). Each processing element inside P_i and N_i has four input channels (three data channels and one logic channel) and one output channel. The instructions executed in the processing elements are shown in Figure 2.

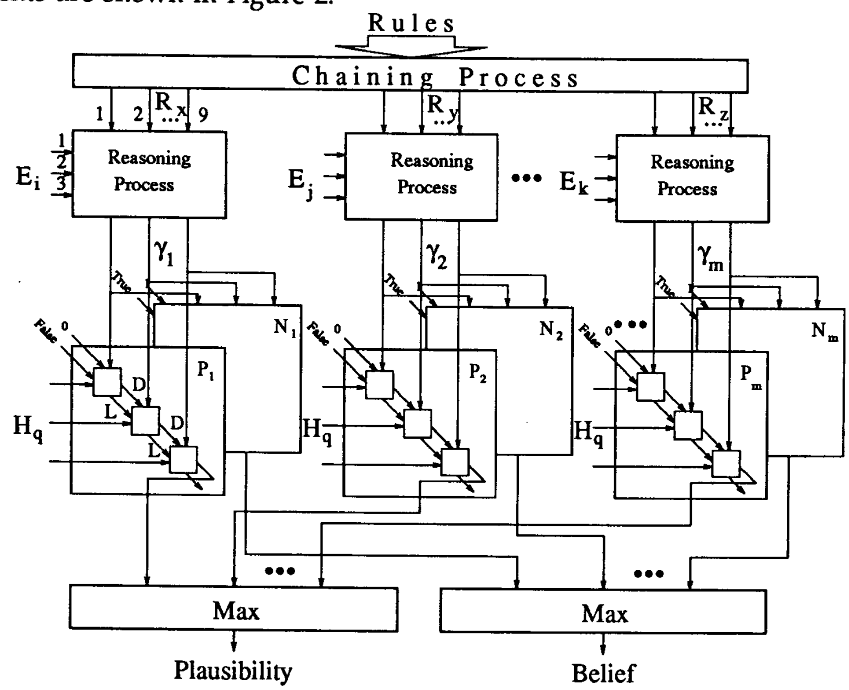

Figure 2. Systolic array implementation of possibilistic evidential reasoning system

4. Fuzzy washing machine reasoning with possibilistic evidential model

For testing the evidential reasoning model architecture a fuzzy washing machine with three rules for selecting the washing powder quantity given the dirt level is considered. The fuzzy rules are as follows:

A: IF dirt ***low*** *THEN use washing powder* ***around 40 to 60 grams.***
B: IF dirt ***medium*** *THEN use washing powder* ***around 70 to 110 grams.***
C: IF dirt ***high*** *THEN use washing powder* ***nearly 200 grams.***

Where the bold faced fuzzy predicates in antecedents and consequents of the rules are fuzzy sets defined on dirt on a hypothetical scale and washing powder in grams scale.

Given a fact that dirt is **'between low and medium'** the credibilities derived for

Instructions executed in a processing element of the N module:	Instructions executed in a processing element of the P module:
Instruction 1: If $\gamma(x) < 1.0$ then temp = 0.0 else temp = 1.0.	Instruction 1: If $\gamma(x) < 1.0$ then temp = 0.0 else temp = 1.0.

Instruction 2:
If $H_q(x) \geq$ temp
then
logic = true
else
logic = false.
	Instruction 2:
	If (Min ($H_q(x)$, temp) = 1.0)
	then
	logic = true
	else
	logic = false.

Instruction 3: maximum = MAX $\{H_q(x), 1.0 - \gamma(x)\}$	Instruction 3: minimum = MIN $\{H_q(x), \gamma(x)\}$.

Instruction 4:
If (logic AND in-logic)
then
out-min = MIN (in-min, maximum)
out-log = true
else
out-min = 0.0
out-log = false.
	Instruction 4.1: (all units except last one)
	If (logic OR in-log)
	then
	out-max = MAX (in-min, minimum)
	out-log = true
	else
	out-max = MAX (in-min, minimum)
	out-log = false.

	Instruction 4.2: (for last P-processing element only)
	If (logic OR in-log)
	then
	out-max = MAX (in-min, minimum)
	else
	out-max = 0.0.

Table 1. Instructions for N and P module processor elements

hypotheses for washing powder {40 to 60 grams, 30 to 70 grams, 70 to 110 grams, 50 to 130 grams} are as follows:

Hypothesis	Credibilities
40 to 60 grams (H_1)	[1, 0.05]
30 to 70 grams (H_2)	[1, 0.65]
70 to 110 grams (H_3)	[1, 0.05]
50 to 130 grams (H_4)	[1, 1].

It can be observed from the derived credibilities that the system hypothesis that includes more of both consequents of Rule A and Rule B are more credible than the others, as the given fact lies between the antecedents of these rules.

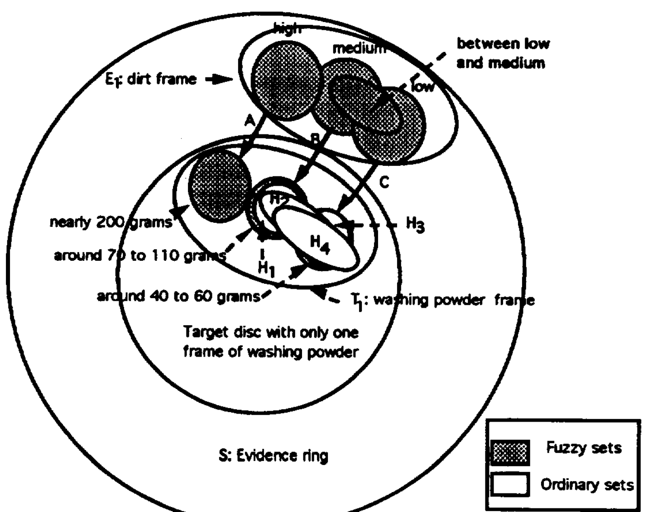

Figure 3. Evidential reasoning model for a fuzzy washing machine

5. Conclusions

Possibilistic evidential reasoning systems are highly compute bound. Different specialised architectures have been suggested for simple *reasoning systems*. Systolic arrays are most suitable for regular and symmetric algorithms. The suggested architecture exploits the regularity in possibilistic evidential reasoning systems to reduce their complexity.

A fuzzy systolic array for the *possibilistic evidential reasoning* system has been described in this paper. The architecture is composed of mainly three modules. The first module implements the reasoning process. The Possibility and Necessity are implemented in the second module. Plausibility and Beliefs are computed in the third module by combining the Possibility and Necessity obtained from the second module. In this paper only the second and third modules of computation has been shown on systolic arrays. These modules are simulated on an IBM PC using the Scheme programming language. To test the algorithms, instructions given in Table 1 are executed in the simulator using 20 processing elements per module. The results obtained from the simulator are found to be correct. Design and implementation of the first module is currently in progress. For the time being in the first part all possible reasoning chains through the rules are considered to be fired. By feeding the evidence (facts) continuously to the first module, Plausibility and Beliefs are obtained continuously for different hypotheses which are fed to the second module. Therefore, this architecture is most suitable for adaptive control systems [13] which operate with fuzzy rules in real-time. The architecture is linearly scalable with increase in the size of the evidence and hypothesis sets.

References

[1] H T Kung., "Why systolic architectures?" *IEEE Computer*, Jan. 1982, pp. 37-46.

[2] M A Manzoul and H A Serrate., "Systolic VLSI array for fuzzy logic in expert systems," *ACM Computer Science conference*, 1988, Atlanta, pp. 380-383.

[3] M A Eshera and S C Barash., "Parallel rule-based fuzzy inference on mesh-connected systolic arrays," *IEEE Expert*, winter 1989, pp. 27-35.

[4] H Tanaka., "A parallel inference machine," *IEEE Computer*, May 1986, pp. 48-54.

[5] M Togai and H Wantanabe., "Expert systems on a chip: An engine for real-time approximate reasoning," *IEEE Expert*, Fall 1986, pp. 55-62.

[6] M Lim and Y Takefuji., "Implementing fuzzy rule-based systems on silicon chips," *IEEE Expert*, Feb. 1990, pp. 31-45.

[7] S M Mohiddin and D I Saunders, "PERFEXT: Possibilistic Evidential Reasoning Fuzzy Expert", *11th European meeting on Cybernetic and Systems Research*, April 1992, Vienna, Austria.

[8] D Dubois and H Prade., *Possibility theory*, Plenum Press, 1988.

[9] J Yen., *Evidential reasoning in expert systems*, PhD Dissertation, Dept. of Electrical Engineering and Computer Science, Computer Science Division, Univ. of California, Berkeley, 1986.

[10] S M Mohiddin., *Possibilistic evidential reasoning methods and their applications*, Ph. D Thesis, Dept. of Computer Science, Queen Mary and Westfield College, Univ. of London, 1992.

[11] S M Mohiddin and T S Dillon., "Learning Hierarchical fuzzy concepts from Neural networks", *Tech. Report*, Dept. Computer Science and Computer Engineering, La Trobe Univ., Victoria, Australia, 1993.

[12] J Pearl., "On evidential reasoning in a hierarchy of hypotheses," *Artificial Intelligence*, Vol. 28, 1986, pp. 9-15.

[13] L Wang and J Mendel., "Generating fuzzy rules by learning from examples," *IEEE Systems, Man and Cybernetics*, Vol. 22, No. 6, 1992, pp. 1414 - 27.